NO. HY1
FEBRUARY 1956

JOURNAL of the Hydraulics Division

PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

VOLUME 82

THIS JOURNAL

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This Journal is published bi-monthly by the American Society of Civil Engineers. Publication office at 2500 South State Street, Ann Arbor, Michigan. Editorial and General Offices at 33 West 39 Street, New York 18, New York. $4.00 of a member's dues are applied as a subscription to this Journal. Application for second-class mail privileges is pending at Ann Arbor, Michigan.

5856-23

JOURNAL

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

CONTENTS

February, 1956

Papers

(Discussion open until June 1, 1956)

JOURNAL

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

DESIGN OF STABLE CANALS AND CHANNELS IN ERODIBLE MATERIAL

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(Proc. Paper 880)

SYNOPSIS

The design of stable channels in erodible material is complex and depends pon the incomplete technical development of open channel hydraulics, sedi-nent transport, and fluvial morphology. Considerable engineering experience nd judgment are required in an approach to this problem. Major factors which must be integrated into the design of a canal are listed and discussed. The development and present method of design of canals are outlined.

Factors causing a change in stream regimen are enumerated and several nethods by which the proper size and shape of channel can be computed are uggested and one example is presented. Practical consideration and field xperience of channel stability problems are listed. An outline of basic data eeded for adequate design of a channel is given.

INTRODUCTION

Irrigation projects require conveyance of water to the land by means of anals and laterals and the removal of the water not consumed by the irrigable reas by means of a constructed drainage system and/or improvement of atural drainageways. Usually, both the irrigation and drainage water must e conveyed by channels in erodible material. Basinwide water resource levelopment and transbasin diversion have radically changed the regimen of ivers and streams so that channel rectification, bank protection, and grade tabilization must be used to protect adjacent property. In all natural drain-geways and some drains, it is essential that the sediment load of the water e considered in the design of the channel cross section and in the improve-nent of the channel.

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ote: Discussion open until June 1, 1956. Paper 880 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol 82, HY 1, February, 1956.

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For canal design, the presence of clay and fine silt requires no special consideration other than that taken for sediment-free water. An attempt is made to maintain as high a velocity as possible without causing scour to prevent deposition. The presence of coarser sediments has not been considered since the canals receive water from storage reservoirs or from diversion works where most of the coarser sediments are kept from entering the canal.

Two important factors must be considered in the design of canals and drains—water salvage and maintenance cost. With water becoming a scarce and valuable resource, it must not be wasted. In canals, various types of linings are considered, because excessive seepage not only wastes water but waterlogs land, making it non-productive. Rectification of river channels is necessary to decrease the water lost in transit caused by high water tables and nonbeneficial consumptive use of phreatophytes. Canals and channels should be designed to attain the least annual total cost and to prevent conditions which would lead to progressive increase in maintenance cost.

Part 1—Canals

This portion of the paper on canal design is presented to outline briefly the background and development of the present practice used by the Bureau of Reclamation in arriving at a usable section for a canal prism. It is expected that the true value of this work may be as much in defining the limitations of present practice as in explaining what the practices are. The background is essential because irrigation systems in this country differ in material ways from those in other parts of the world. Until these basic differences are evaluated, it is not apparent why rules, formulae, and equations apparently acceptable elsewhere have proved unusable in our work.

It is possible to develop a stable channel in erodible material where the capacity and silt load are constant. It is also possible to develop a channel that will erode and refill during certain seasons so that at any time during 1 year, the channel will be in the same state as in the previous year. However, neither of these conditions has a wide practical application, and it is this failure of theoretically perfect channels to function properly in practice which gives rise to the difficulties of canal design.

It is necessary that the field conditions which render impractical a highly theoretical approach be realized so that the engineer designing a canal section can properly evaluate the usefulness of his theoretical computed section. The factors which influence channel choice and which must be integrated into the design by experience and judgment are so numerous that only a few major items will be listed.

1) The terrain traversed by a canal may extend for over a hundred miles. In such distance, soils vary widely. The major types such as silts, sands, clays, broken rock, or solid rock can be considered separately. However, the soils in any class also vary radically within themselves so that assumed channel factors do not apply from section to section. This is clearly shown in loessal areas.

2) The soil in place may be stable against scour but when worked in excavation it becomes easily erodible. This would indicate that the water prism should be in undisturbed earth. However, the increased cost of such a location is obviously not justifiable and the alinement chosen is dependent on an evaluation of costs of various possibilities and types of equipment that

may be used. The development of earth-moving equipment itself has changed design. A bank built with excavating and hauling equipment is considerably different from that placed by draglines. These factors influence the selection of the design section and must be considered in determining the canal side slopes and alinements.

3) After construction, the canal must not only be intrinsically stable but obviously must also hold water. Leakage results in considerable water loss, and it may ruin land by water logging. Moreover, excessive losses require larger sections to provide for deliveries in addition to loss. Hence, lining is a major consideration. There are several types of membrane linings which require earth cover to afford protection against excessive scour. The selection of the type of lining requires a study of costs among the various possibilities for a given section. An adequate theoretical approach is very valuable for this comparison.

4) Some scour can be tolerated in cut but where extensive reaches must be in fill, scour cannot be allowed. Hence, judgment must dictate the "safety factor" against scour that is to be used.

5) A knowledge of the land use and development is also necessary. A new project will result in years of relatively low demand and may never fully utilize the capacity provided. On the other hand, a developed project may require heavy use of sections almost as soon as the new supply canal is constructed. A new system can, therefore, safely carry a higher scour potential if serving an undeveloped area than it can in serving a developed area because complete seasoning is possible before full use will be required.

6) Irrigation operations also influence designs. Is the canal to be in service consistently or will it be empty during winter months? Are weed problems the primary maintenance item or will blow sand tend to plug the section? Will the canal be checked up for side deliveries at low flows or will the flow seek its own level for various demands? All of these must influence the section and be properly evaluated if the minimum overall project cost of construction plus operation plus maintenance is to be realized.

The above factors are listed to illustrate design problems that will alter the theoretically perfect section, if known, which is rarely the case. The factors we do not know will be brought into the discussion of our theoretical design approach as they occur.

The studies made for a stable channel in the All-American Canal embraced the initial approach to the problem of design of stable channels. Previous data and formulae were studied but the application seemed beyond their possible extension. The canal would be forced to carry clear water while the reservoir filled in, then probably a considerable silt load. Further, it traversed many miles of desert terrain of widely varying soil conditions. After study of available material and data, it was decided to install clarifying devices to maintain a clear water canal and design accordingly.

The shearing forces on sides and banks were evaluated and the main canal designed with a B/D of approximately 8 to 1 with 1-3/4 to 1 side slopes and a maximum velocity of 3.75 feet per second. This section is still stable after 16 years of almost constant use. Naturally, such a channel gives rise to the question of whether it could have been smaller and still be stable.

Results of our past experience have been tabulated showing maximum and minimum hydraulic properties, depending on soil types.(1) These data serve

as a guide to the designer in establishing proper canal sections. To guard against water loss or land water-logging, lining of all types has been used. With few exceptions, these linings are resistant to scour and hence do not require study of erosion but have a series of problems of their own. The question of sediment transport in these and in erodible channels will be discussed in the second part of this paper. Hard surface linings are very often used in noncohesive soils and hence limit the number of canals that are available for study of erosion criteria in these types of soils.

Between 1948 and 1950, a program of investigation was formulated and under the direction of Mr. E. W. Lane put into effect. The basic approach was the extension of the Tractive Force principle. The results of a great number of independent developments and equations were compared to each other and to our canal sections with very little correlation between formulae or with field conditions being found. These results were then analyzed, using tractive force for evaluation, and a better agreement seemed obtainable although the spread of answers was still too wide for practical application.

Tractive force is simply the pull of water on the wetted perimeter. For an infinitely wide channel, this value on the bottom is:

TF = wds where
TF = tractive force in pounds per square foot
w = weight of water in pounds per cubic foot
d = depth in feet
s = slope of hydraulic gradient (tangent of angle with horizontal)

The Bureau of Reclamation's Hydraulic Laboratory Report No. HYD-352, "Progress Report on Results of Studies on Design of Stable Channels," gives a full explanation of this analytical work together with the influences of sides of different slope and base widths. Reports were also prepared developing the theoretically perfect shape and evaluation of critical tractive force.[2,3]

Laboratory Report No. HYD-352 contains charts from which various B/D ratios and corresponding side slopes can be evaluated for design use. It is necessary to know the mechanical analysis of the soil being investigated. The effect of cohesion cannot, as yet, be accurately evaluated, but studies are in progress to improve our understanding of this factor. This report computes, for a given soil, the minimum section and is usable as a basis for design. Physical limitations of survey work and excavating machinery prohibit the use of the section as computed. Also, operation roads and structure design alter the shape. However, a realization of the theoretical minimum section is of value to the designer. Laboratory Report No. HYD-366 is a theoretical evaluation of the maximum limit of tractive force as applied to any soil particle and provides a better understanding of the mechanics of this approach. All three reports are available from the Bureau of Reclamation, Denver Federal Center, Denver, Colorado.

As the reports explain, the equations do not answer a great many questions. The basic principle seems correct but boundary layers and the turbulent losses certainly need further study and analysis to improve the basic equations. The reports contain laboratory and field research data obtained prior to 1952. This investigation is still continuing on a reduced scale, but several canals have been designed to check certain phases.

The current investigations cover three principal phases which are: Application of the Tractive Force analysis to channels in noncohesive soils,

development of procedures and equipment to determine cohesive properties of the in-line soils, and investigations of soil properties not defined by either the Plasticity Index or mechanical analysis.

The application of the design procedure is being checked by using three reaches of a canal in almost identical soil consisting of fine wind-blown sands and silts. One reach utilizes a tractive force of 0.033 pound per square foot. Others are progressively higher up to 0.075. The higher values are expected to scour, so sections were selected where scour could be controlled by checking if it occurred. These sections have not, as yet, been operated at full capacity.

The effective value of cohesion is a very troublesome problem. A natural tendency to reduce section for economy is an inducement towards overdependence on cohesion with probable resultant high maintenance costs. A reasonable value is obviously needed and there exists, to our knowledge, no device or method of evaluation. Our present approach is to develop an apparatus similar to a vane borer to measure shear resistance, but this has many uncertainties. Another is the use of a submerged controlled jet to give comparative values, and the third is saturated shear values from a loading machine or correlation with plasticity index. So far, progress has been too limited to judge the efficiency of any of these steps.

It has been noticed that certain canals scour while others under very similar conditions do not. This similarity is in width, depth, side slopes, and velocities. It even extends to reasonable freedom from sediment load, mechanical analysis, and plasticity index of soils. Since these usual factors do not account for the dissimilarity of results, soil samples are being taken for chemical examination. It may show that an ion exchange between water and soil hydration of soil material is providing a binder in some localities. Should this prove a dependable condition, future evaluation of a proposed section should include whatever advantage can be taken of such benefits.

Should these steps prove to be fruitful, there will still remain a tremendous field of investigation before theoretical design can predict with certainty a stable channel; many factors will still remain to judgment. However, even in its present stage of development, the use of tractive force seems to provide a useful tool and as accurate an approach as has been devised.

Our design now consists of selecting, from experience and judgment charts, an approximate section which is used in preliminary surveys. For this alinement, geological investigations are made. This consists essentially of a series of test holes and samples. The soil is classified and analyzed for sizes of particle and plasticity. Shear tests are also carried on to be used in determining bank stability.

With these data, the designer investigates the section proposed by applying the tractive force analysis to determine probable stability by reaches and to determine the minimum section that appears usable. Up to this point, the analysis is theoretical. This proposed section must then be evaluated for the effect of cohesion, which is still beyond an exact analysis. It is further considered in relation to cost of original construction, rights of way, risks involved should some scour occur, the desirability in small sections of a scouring velocity to move wind-blown sand, section changes, structure costs, and the advisability of lining. All of these factors will, to varying degree in different projects and areas, influence the section selected.

In short, even with an improved theoretical approach to the design of a stable section, the existing shortcomings of our analysis, practical

consideration, economic conditions, bank stability, and operation and maintenance practices all combine so that, at best, the analysis is a guide and will not, in the foreseeable future, supplant experience and sound engineering judgment in the design of canals.

Part II—Channels Transporting A Sediment Load

The design and rectification, by the Bureau of Reclamation, of channels carrying sediment-laden water is a major problem.

Natural streams can be divided into two main classes, perennial and ephemeral. A perennial stream is considered one in which continuous flow is maintained for long periods of time even though it is dry for some periods. In contrast, an ephemeral stream is one in which flow is directly related to storm runoff and does not flow for periods greater than 24 hours.

The stability of sediment-carrying perennial streams may be upset by (1) decreasing the flow, (2) increasing the flow, (3) increasing the sediment load, (4) decreasing the sediment load, or (5) regulation of the flow. These changes have been found occurring due to the following conditions:

1) Decrease in flow. The major problem encountered by a decrease in flow occurs by diversion of the clear flows as they emerge from the mountainous areas. Because these flows were historically mixed with highly sediment-laden water from high sediment-producing portions of the drainage, a serious problem results when they are diverted. What actually takes place is an annual channel flushing as a part of the channel regimen. Many of the streams draining the Rocky Mountains present this problem. Examples are the South Platte, Arkansas, and Rio Grande.

2) Increase in flow. Diversion of water to a stream from another drainage basin may often increase the flow by an amount greater than the original flow. Some of the transmountain diversion schemes present this problem. Also, off-channel storage reservoir installations may use a small channel for conveyance of releases back to the main channel. Return flow from an irrigable area will often accrue to a small channel draining the area.

3) Increase in sediment load. Changes in agricultural practices and land use have been one of the largest factors encountered in increasing the sediment load of a stream. The sediment load of a stream may be increased by adding highly sediment-laden waste water from an irrigable area.

4) Decrease in sediment load. The sediment load of a stream is decreased by storage reservoirs in a stream system. Clear water releases from a reservoir that is storing highly sediment-laden water very often develop a degradation problem downstream.

5) Regulation of flow. A storage reservoir particularly for flood control may change materially the flow pattern or the flow duration relationships. This may change the discharge that has been most influential in establishing the stream regimen. For example, floods that originally were conveyed by large overbank areas may after storage be released at bankfull capacity for a considerable period of time. The stream regimen may not have been established by long periods of bankfull flow.

The stability of an ephemeral-type stream is presenting one of the most acute channel problems with which the Bureau is faced. The problem appears

to be different than that of perennial stream channels and also the more sensitive. This may be due, in part, to the more divergent changes that upset the stability. The factors that upset the stability of ephemeral channels are much the same as for perennial streams, but the channel reactions appear to be different and, for that reason, are mentioned separately. The factors are (1) increasing the flow, (2) regulation of the flow, (3) increasing the sediment load, or (4) decreasing the sediment load.

1) Increasing the flow. Project development is often responsible for an increase in flow of an ephemeral channel and may change the flow from ephemeral to perennial. Ephemeral channels may be used as carriers of transbasin diversions, conveyance of water to and from off-channel storage reservoirs, and as canals in project distribution systems. The flow in small ephemeral channels is often increased when storm flows are rerouted from their natural courses to protect irrigable areas, canals, structures, etc. Irrigation wastes are often dumped into ephemeral channels to be conveyed back to a main stream channel.

2) Regulation of flow. The flow pattern of an ephemeral channel can be so changed that it will affect the stability of the channel. Storm detention in either mainstem dams or a number of tributary dams may change the flow pattern to the extent that flows may appear in the channel for several days, whereas under natural conditions the flow was a rush of water. Also, water that under natural conditions appeared as overbank flow may be confined to the channel and in many cases at near channel capacity. Conservation measures on the drainage area, such as contour farming, contour terracing, and changes in land use and cropping pattern may be effective in changing the flow pattern.

3) Increase in sediment load. An increase in sediment load can be both from upstream channel failures and production from the drainage area. Increases in load from the drainage area usually occur due to changes in the drainage area such as changing forest areas to cleared areas, forest burn areas, pasture land to cultivated land, and the development of irrigation areas. Surface wastes from irrigable areas often contain sediments.

4) Decrease in sediment load. When the sediment load of a stream is decreased, there is less transport capacity required of the stream to move the sediment. The difference in the sediment load may be made up in part from the bed and banks if the proper material is available from this source. A decrease in sediment load can occur due to storage in detention dams and due to drainage area conservation measures.

The Bureau has the problem of designing artificial channels to convey water with sediment loads. These channels are generally not associated with irrigation distribution systems, but are channels that convey diverted stream flow to off-channel storage reservoirs and river conveyance channels such as the salvage of water in the delta area above Elephant Butte Reservoir. Heavy loads of sediment usually cannot be prevented from entering project drains when storm flows are carried. These drains may be either perennial or ephemeral. The design of a channel to carry sediment loads requires that certain basic data be available. The data desired are as follows:

1) A flow-duration curve of the flows the channel will have to carry.

2) A sediment-rating curve showing the average sediment load in tons per day for a given discharge that the channel will have to carry.

3) Size-distribution curves showing average texture of the sediments at various instantaneous discharges throughout the range of the sediment-rating curve.

4) Representative bed material size-distribution curve above the potential diversion point.

5) Slope of the channel above the potential diversion point.

6) Representative material size-distribution curve for the material through which the channel is to be constructed. Variations in texture with depth and plasticity index if applicable.

7) The slope of the proposed channel and limitations upon variation.

8) The type and vigor of the existing native vegetation in the area through which the channel may traverse.

9) Outline of any limiting conditions upon the channel design and future operation.

A method that has been used by the Bureau in developing a channel design is the transport method.(4) Essentially, it is a cut-and-try approach with transport computations made until the point is found where the transport capability of the channel matches that required by the sediment load the channel will have to carry. Only the sediments coarser than 0.0625 mm are considered in this type of an analysis. Such a computation was performed for the conveyance channel above Elephant Butte Reservoir using the Einstein "Bed-Load Function for Sediment Transportation in Open Channel Flows."(5) In this case, the slope was that of the valley and very little variation from the natural was practical. The bed material of the channel from which the water was diverted was assumed as what could be expected in the proposed channel as the native material and the slopes were comparable. With a given slope and bed material, there was computed the transport capability in pounds per second per foot of width by size fractions (0.0675 mm, 0.120 mm, 0.125 mm, 0.250 mm, 0.50 mm, etc.) for various hydraulic radii that might be expected. The results were plotted on semilog paper (transport as ordinate, log scale and R abscissa, coordinate scale) and transport curves drawn for each size fraction, Figure 1. Using the Einstein Bed-Load Function computations, Figure 2, there were developed discharge-hydraulic radii relationships for channels of various widths. These relationships were expressed by curves with hydraulic radius as abscissa and discharge as the ordinate, Figure 3. Table 1 was next set up which essentially represents a mechanical integration of the flow-duration curve for the flows the channel will be expected to take. It may be noted that some of the coarser fractions do not show adequate transport. The total sand transport was used rather than the individual fractions because of the assumptions necessary and the limitations of the formula to accurately compute the transport of the individual size fractions. The sediment load was then computed for various widths of channel using the curves described. The sediment load computed for the various widths was plotted against the channel width. This plot was on semi-log paper with the sediment load in tons per day plotted on the log scale and the channel width plotted on the coordinate scale, Figure 4. The plot of the points defined reasonably well a straight line. From this relationship the width was determined at which the average sand sediment load expected will be transported.

The width computed above was not constructed due to limitations of earth-moving equipment. A narrower width channel was constructed and is being allowed to erode. A representative curve of widening rate was determined

using the transport curves. The curve of widening rate appears to approach asymptotically the optimum width computed. The width to which the channel is expected to erode is where the widening rate curve breaks and starts to approach a straight line. It was thought the narrower width chosen would become stable because of bank-restraint barriers offered by the vegetation in the area.

The approach can be employed using several of the other acceptable sediment transport functions. The approach has three weaknesses that preclude the results as an exact computation. The bed material size is an influential component of the computation, and this variable is assumed throughout. The width of compatible transport may still have an erosive force that will produce bank failure and therefore stability is not assured. Our experience shows that existing transport functions and formulas have not been developed to a point of exact determination of sediment movement.

Another method that has been used by the Bureau is the Maddock-Leopold principle. The general theory of this approach was presented in Geological Survey Professional Paper 252, titled, "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications," by Thomas Maddock, Jr., and Luna B. Leopold. The formula that has been used by the Bureau is not that presented by the authors in the paper, but is the result of subsequent work. Also, the formula may not be the latest thinking by the authors but when adapted showed a significant correlation coefficient with observed field conditions to warrant consideration for practical applications. The formula is as follows:

$$w/d = \left(\frac{S^{1/2}/n}{.225(C_s V_s)^{.395}} \right)^3 Q^{.555}$$

where:

- w = width of the water surface in feet
- d = mean water depth in feet
- S = slope of the hydraulic gradient
- n = rugosity coefficient
- C_S = sediment concentration in ppm by weight
- V_S = mean fall velocity of the sediments in feet-per-second at the appropriate temperature
- Q = the dominant discharge in feet-per-second

The w/d ratio can be resolved into separate components by application of the Manning equation. For the wide shallow channels, the Manning equation can be resolved to

$$Q = 1.486\ (S^{1/2}/n)\ wd^{5/3}$$

by letting the mean depth equal the hydraulic radius and the area equal the product of width and depth.

The term "dominant discharge" has been used by many authors and has been defined several ways. It has been recognized by students of stream morphology that a natural stream channel does not maintain a constant

unwavering condition of stability. At one discharge, the channel may be degrading, at another aggrading, at one widening, and at another narrowing; in fact, all these conditions may be found in action in a stream at the same time within separate reaches, but the overall effect is not one of continued aggradation, degradation, widening, or narrowing. The dominant discharge is herein defined as that discharge most influential in dictating the general condition of stability. In Bureau usage, it has been defined quantitatively as the discharge that will carry the greatest sediment load coarser than 0.0625 mm with respect to time. Generally, it will be found slightly greater than the median discharge. The Maddock-Leopold procedure, as described above, has three weaknesses that prevent the results from being conclusive. The empirical correlation was developed from data obtained mostly from natural streams, whose shape has been developed through long periods of time. Therefore, the channel shape computed may be that which the channel will trend to and will attain in time. Due to natural perimeter barriers such as heavy cobbles, rock, vegetation, and highly plastic soils, the time to attain the width may be beyond our economic consideration. Another weakness may be seen by examination of the formula. If the $C_s V_s$ value approaches the condition of clear water, then the w/d ratio becomes larger than would be reasonably expected. Also, the rugosity coefficient, "n," of the design channel as affected by variable sediment movement cannot be accurately predicted.

Another empirical approach used is that by T. Blench as outlined in his book, "Hydraulics of Sediment-Bearing Canals and Rivers," copyright 1951. Considerable judgment and experience are required in the application of the Blench approach, and poor judgment is likely to give corresponding results.

Part I of this paper presents the Tractive Force principle as developed by E. W. Lane. The Tractive Force principle does not take into account the influence of sediment loads, particularly heavy loads of coarse material. However, it does give a factor of bank resistance in noncohesive materials. Also, the report does not give a good definition of allowable tractive forces for cohesive materials, the influence of vegetation, or for the ephemeral flow condition. Field investigation has been made of about 70 artificial and natural channels with varying conditions of cohesiveness, vegetative growth, and for both perennial and ephemeral flow. Most of the channels have 20 years of service or more and were selected as effectively stable, although some erosion may occur at times. Rough working curves have been developed and are now being checked with field experience.

In working with channel-stability problems, several observations of channel phenomena have influenced the approach to the problems. A few of these are as follows:

1) There are conditions where the sediment load of a stream exceeds the transport capability, yet serious bank erosion will occur.

2) Continuity of tractive force and smooth hydraulic transition are very often a requirement for stability. High tractive forces in one reach may erode bed and bank material that will deposit in reaches of low tractive forces, and aggradation will result. Constrictions such as natural narrow rock controls and inadequate bridge openings may cause backwater and eddy conditions which produce aggradation areas above and scour areas below. The upstream area can often be the more serious when coarse materials deposited during flood stage force the lower flows to vulnerable bank areas.

3) The allowable tractive forces for ephemeral streams in cohesive soils are higher than perennial streams in the same soil. Erosion can usually be expected, particularly at the historical flood magnitudes even when only small amounts of continuous flow are added.

4) When large amounts of low flow water are added to meandering streams sufficient to create heavy bend erosion, the pattern of meander movement is upset. When floods are no greater than in the past, the deposition on the inside of bends is not as fast as the accelerated bend erosion. The end result is a loss of the meander belt, increased slopes, increased scour ability, and general channel failure.

5) Vegetation is effective in increasing the allowable tractive force on both the bed and the banks. However, vegetation will not grow in the perennially inundated portions of the channel unless velocities are exceedingly low.

6) Sediments will not follow the water into the underlying gravels of an influent stream. Influent streams that carry large amounts of sediment are special stability problems.

7) Sediments finer than 0.031 mm generally do not seriously affect channel stability problems except for side berming action in areas of heavy grass vegetation.

CONCLUSION

The approach to channel stability problems is one that requires the application of all that is known about river hydraulics and sediment transport. It has been the Bureau of Reclamation approach to use all of the approaches herein described in a combination. For example, the Maddock-Leopold and Blench procedures will give a width at the dominant discharge. This width is then checked to see if the sediment will be transported. If this checks, then the bank tractive forces are checked. If the bank tractive forces are too high for the bank material to resist, then additional barriers such as riprap, vegetation, and revetments are needed. If the bank and bed material show considerable more allowable tractive force, then the channel will remain stable at a narrower width as dictated by the bank and bed allowable tractive force. Transport will almost always be greater than required with the narrower widths. At the same time, one must always check the seven special conditions mentioned above. It is expected more of these special conditions will be added to the list with experience.

The problem of stable channels is, in our opinion, one that is complex. It appears to be a type of engineering that does not lend itself to precise design and is dependent upon the judgment, experience, and skill of the engineer. It is felt by Bureau engineers that the people faced with these problems should strive cooperatively to develop procedures that point the way to simpler and more conclusive solutions. Many more problems face us in the future than have required attention in the past as water resource development continues.

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2. Bureau of Reclamation Laboratory Report No. HYD-325, "Stable Channel Profiles."

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4. "Conveyance Channel Widening Study—Middle Rio Grande Project," Bureau of Reclamation Report, by J. M. Lara and C. R. Miller.

5. U. S. Department of Agriculture Technical Bulletin No. 1026.

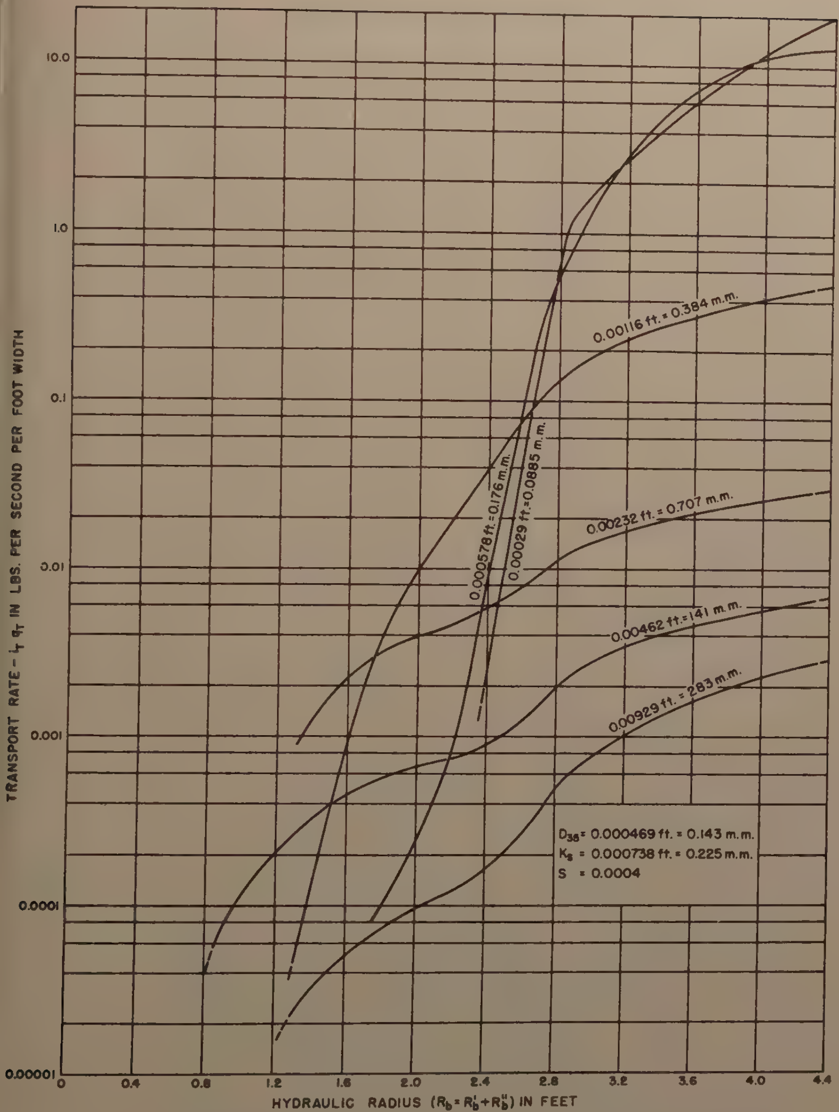

FIG. I - UNIT SEDIMENT TRANSPORT ($i_T q_T$)
FOR VARIOUS PARTICLE SIZES - CONVEYANCE CHANNEL

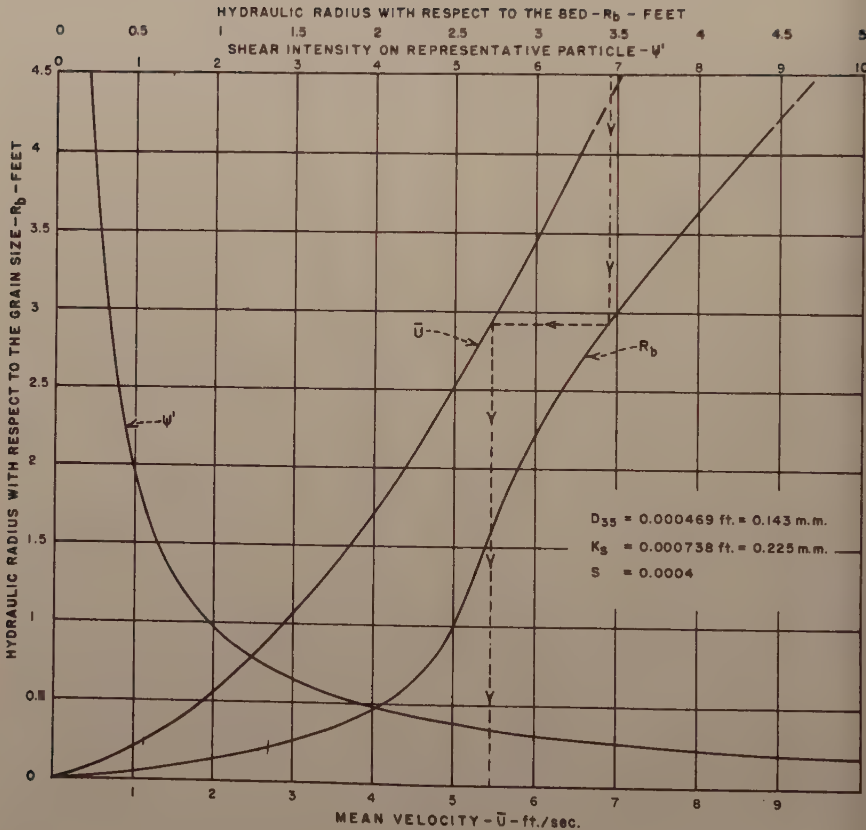

FIG. 2 - EINSTEIN CHANNEL CHARACTERISTICS
FOR CONVEYANCE CHANNEL

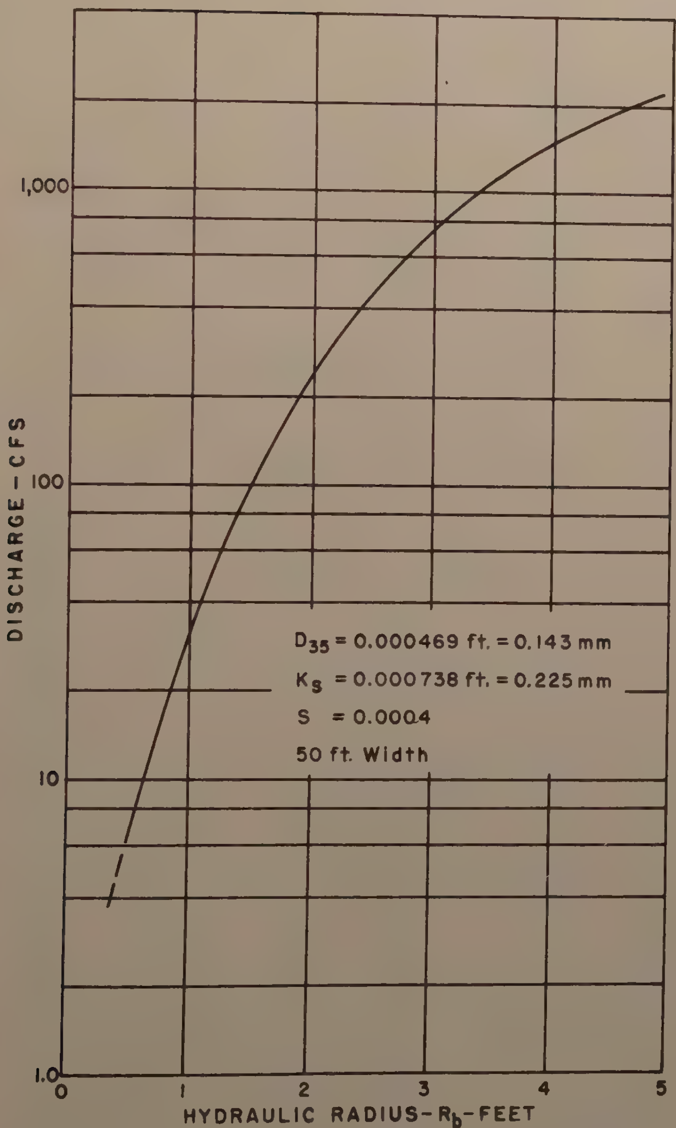

FIG. 3 - DISCHARGE AND HYDRAULIC RADIUS FOR CONVEYANCE CHANNEL

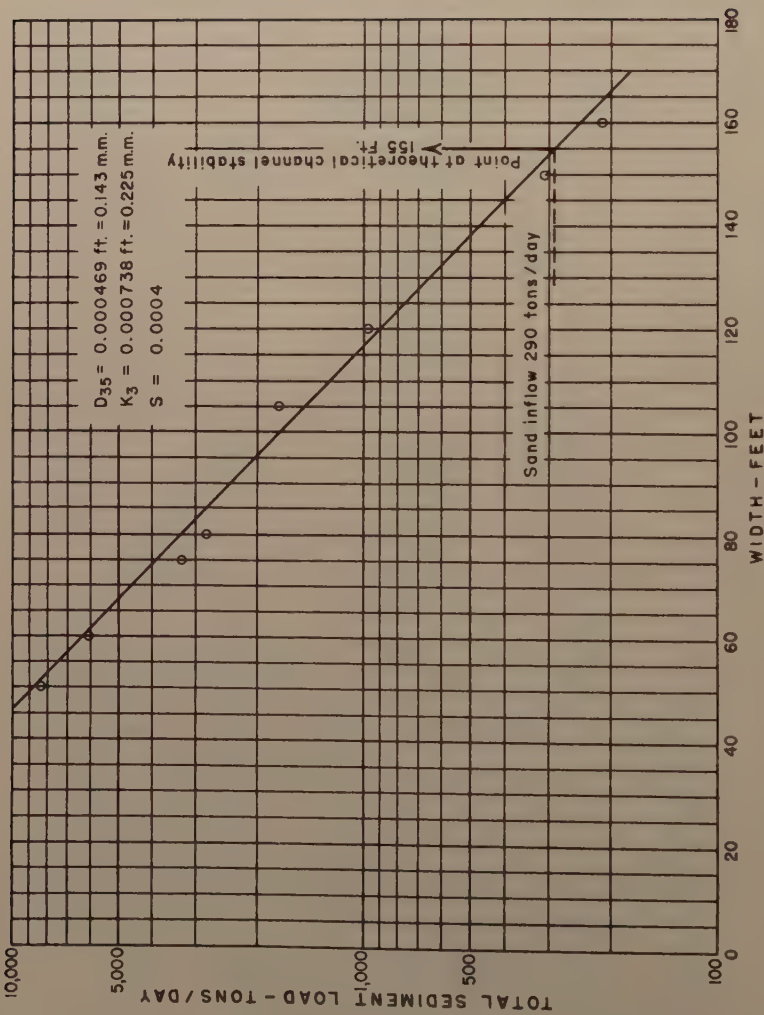

FIG. 4 - RELATION OF SEDIMENT TRANSPORT AND WIDTH FOR CONVEYANCE CHANNEL

TABLE 1

Transport Load for Conveyance Channel
Middle Rio Grande Project
Rectangular Channel - 50-foot width, S = 0.0004

Q_w	Flow Duration Percent	R_b	P_b	Size of Material--ft. X 10^{-6} 290	578	1160	2320	4620	9290	Pounds Per Second Per Foot	Tons Per Day Per Foot	Tons Per Day
1.4	1.0	0.13	50									
11	4.0	0.67	50									
78	10.0	1.38	50			0.000190 0.000821	0.001200 0.005184	0.000310 0.001339	0.000032 0.000138	0.001732	0.007482	0.374
244	10.0	2.03	50		0.000285 0.001231	0.014000 0.068480	0.004050 0.017496	0.000680 0.002938	0.000103 0.000445	0.019118	0.090590	4.530
415	10.0	2.43	50	0.00400 0.01728	0.01350 0.05832	0.04500 0.19440	0.00610 0.02635	0.00096 0.00415	0.00018 0.00078	0.06974	0.30128	15.064
580	10.0	2.74	50	0.2800 1.2096	0.3950 1.7064	0.1160 0.5011	0.0100 0.0432	0.0017 0.0073	0.0004 0.0017	0.8031	3.4693	173.465
720	10.0	2.97	50	1.63000 7.0416	1.19000 5.1408	0.17900 0.7733	0.01410 0.06091	0.00268 0.0116	0.00071 0.0031	3.01649	13.0313	651.565
840	10.0	3.13	50	2.51000 10.8432	2.10000 9.0720	0.21900 0.9461	0.01600 0.06912	0.00323 0.0140	0.00093 0.0040	4.84916	20.9484	1,047.220
1080	10.0	3.46	50	5.30 22.90	5.30 22.90	0.291 1.26	0.0201 0.0868	0.0043 0.02	0.00143 0.01	10.91683	47.18	2,359.000
1500	7.0	4.02	50	13.5 40.824	13.5 40.824	0.42 1.270	0.026 0.079	0.0057 0.017	0.00232 0.007	27.45402	83.021	4,151.050
										Total		8,402.27
T/D for Q_w below 2000				4141.68	3985.14	250.71	19.40	3.92	1.36	. . . .		8,402.21
T/D for sand inflow < 2000				0.02	6.38	131.47	108.47	32.62	10.58			
			Diff.	4141.66	3978.76	119.24	minus -89.07	minus -28.70	minus -9.22			8,112.67

JOURNAL

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

TABLE OF CONTENTS

DISCUSSION
(Proc. Paper 881)

Discussion of
"SURFACE-WATER SUPPLY FOR IRRIGATION IN THE VERMILION RIVER BASIN, LOUISIANA"

by E. L. Hendricks
(Proc. Paper 489)

E. L. HENDRICKS,[1] A.M. ASCE.—The author is indebted to Mr. Oakes for the background and supplemental information provided in his discussion of the writer's paper. It was the purpose of the original paper to present certain data and such analyses of that data as are pertinent to a description of the water-supply problem in the Vermilion River basin. Mr. Oakes has properly gone a step further to discuss the relation of these facts to the planning and design of facilities to alleviate the recurring water shortages in the Basin.

Mr. Oakes' observations on a possible shift from ground water to surface water as a source of supply for irrigation, in the event an adequate supply of surface water is made available, are worthy of comment. Although limited access and other reasons would not likely permit all ground-water users in the Vermilion River basin to change to the use of surface water, the probability of a significant shift in water use warns that in planning the engineering works the probable effects on both ground and surface water supplies must be considered.

Mr. Oakes' conjectures on the factors affecting the reclamation of preseason runoff from Vermilion Bay are interesting. As he suggests, however, these questions can be answered with certainty only by means of a model study. It is the writer's understanding that the Louisiana Department of Public Works who sponsored the investigation reported in the writer's paper, is now sponsoring such a model study as a preliminary phase in the planning of works to alleviate the water shortages in the Basin.

1. Staff Engr., Technical Coordination Branch, USGS, Atlanta, Ga.

Discussion of
"THE LOG-PROBABILITY LAW AND ITS ENGINEERING APPLICATIONS"

by Ven Te Chow
(Proc. Paper 536)

VEN TE CHOW,[1] A.M. ASCE.—The writer is deeply indebted to the discussers for their valuable contributions and constructive comments which have added tremendously to the value of the paper. He wishes also to take this opportunity to express his appreciation to those who, owing to the limitation of time, were unable to prepare a comprehensive discussion, but have forwarded their brief comments by direct correspondence. Some high lights of these comments are incorporated in this closing discussion. The diversity in nature of all the discussions makes it desirable to present this discussion under various headings.

More References on Studies and Applications of the Law

Mr. Morton S. Raff[2] drew the writer's attention to a mathematical presentation of the logarithmic normal distribution by Hald.(67)[3] Prof. E. W. Lane[4] emphasized the fact that in the field of the study of the sediment load of streams, logarithmic probability plotting is extensively used to represent the particle size of the sediments involved. In this connection he added two references: one by Krumbein and Pettyjohn(68) and the other by Otto;(69) and the writer also found other related references by Krumbein(70)(71) on size frequency distribution of sediments and the reference by Loveland and Trivelli(72) on size frequency distribution of particles in photographic emulsions. For applications in meteorology, the writer has read a discussion of the log-normal distribution in the Handbook by Brooks and Carruthers(73) and from this source he learned that the first attempt to fit a normal distribution to the logarithms of values of a meteorological element appears to have been by Backhouse(74) in 1891. In connection with flood studies more references are found on the use of logarithmic plots of runoff statistics in Glen Cannich, Scotland by Wolf(75) and on a generalized logarithmic normal distribution discussed by Thom.(76) The paper by Beard,(77) same as,(46) and its discussions by Thom, Benson and Rantz, and Alexander,(78) as pointed out by Mr. Sammons, are additional information on the recent development of log-

1. Research Assoc. Prof. of Hydr. Eng. and Member of the Teaching Faculty of the Graduate College, Univ. of Illinois, Urbana, Ill.
2. Mathematician, Highway Transport Research Branch, U. S. Bureau of Public Roads; by correspondence dated March 10, 1955.
3. Numerals in parentheses refer to corresponding items in the Additional List of References. The numbers are continued from the List of References appended to the paper.
4. Prof. of Civ. Eng., Colorado A. and M. College, Ft. Collins, Colo.; by correspondence dated December 10, 1954.

probability studies of floods. For a comprehensive mathematical treatment of the logarithmic transformation of the normal probability distribution, Chow(79) has presented elsewhere a detailed procedure on the determination of the frequency factor in log-probability plotting.

The Central Limit Theorem

Messrs. Isadore Enger,[5] Raff,[2] and Alexander and Karoly have called attention to an error in the paper, under the heading "Theoretical Interpretation of the Law," resulted from the missing of the words, "square of" in front of "standard deviation." Since "the square of standard deviation" is known as "variance," the correct statement is:

> "Whatever be the distribution of the independent variates X_r, subject to certain very general conditions, the sum $X = X_1 + X_2 + \ldots + X_r$ approaches, as r increases, the normal distribution whose mean and variance are respectively equal to the sum of the means and the sum of the variances of the variates."

Mr. Goodrich suggests that for the benefit of practicing engineers the statement of the central limit theorem be amplified and more fully explained. This is a fine suggestion since the theorem is not only of paramount importance to mathematicians, but also very useful to engineers for various applications. Prof. Mood(80) once said:

> "It (the central limit theorem) is the most important theorem in statistics from both the theoretical and applied points of view. And it is one of the most remarkable theorems in the whole of mathematics."

A working knowledge and thorough understanding of the theorem should prove beneficial to practicing engineers. Without the involvement of the advanced mathematics, a more fully explained statement of the theorem is attempted as follows:

In the above revised statement of the central limit theorem, X_1, X_2, X_3, $\ldots X_r$ represent variates which are independent of one another and each of them has its own distribution. For instance, the variate X_1 may be composed of a population of ${}_1X_1$, ${}_2X_1$, ${}_3X_1$, . . . , and the distribution of this population has a mean $\overline{X}_1$ and a variance σ_1^2. Similarly, the other variates: X_2, X_3, $\ldots X_r$ have the corresponding values of mean: $\overline{X}_2, \overline{X}_3, \ldots \overline{X}_r$, and values of variances: $\sigma_2^2, \sigma_3^2, \ldots \sigma_r^2$.

Now, from the population corresponding to the variate X_1, elements ${}_1X_1$, ${}_2X_1$, ${}_3X_1$, . . . are drawn at random, and similarly for other variates. Table D-1 should help to show these elements being drawn at random from the populations.

Referring to the revised statement of the theorem, the sum of the variate is

$$X = \sum_{r=1}^{r=r} X_r = X_1 + X_2 + X_3 + \cdots + X_r$$

5. Statistician, U. S. Weather Bureau, Washington, D. C.; by correspondence dated March 30, 1955.

Then, the sums for the elements drawn at random from the populations are

$$_1X = \sum_{r=1}^{r=r} {}_1X_r = {}_1X_1 + {}_1X_2 + {}_1X_3 + \cdots + {}_1X_r$$

$$_2X = \sum_{r=1}^{r=r} {}_2X_r = {}_2X_1 + {}_2X_2 + {}_2X_3 + \cdots + {}_2X_r$$

$$_3X = \sum_{r=1}^{r=r} {}_3X_r = {}_3X_1 + {}_3X_2 + {}_3X_3 + \cdots + {}_3X_r$$

and so on. These sums are shown in the right-hand side column of Table D-1. It is apparent that the sum X has its population from which the elements resulted from drawing original elements at random are $_1X$, $_2X$, $_3X$, . . . The mean and the variance of this population are $\bar{X}$ and σ^2. According to the central limit theorem, if r increases indefinitely or it becomes very large, then the sum X, as a variate, will have approximately a normal distribution; and the mean and the variance of this distribution are respectively equal to

$$\bar{X} = \sum_{r=1}^{r=r} \bar{X}_r = \bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \cdots + \bar{X}_r$$

and

$$\sigma^2 = \sum_{r=1}^{r=r} \sigma_r^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots + \sigma_r^2$$

The astonishing thing about the theorem is the fact that nothing is said about the form of the population distribution of each original variate. Whatever the form of the distribution, provided only that it has finite values of mean and variance, which is true in almost any practical situation, the sample mean will have approximately the normal distribution for large samples.

The above discussion is made only for a general case. In the special case where the variates all have the same distribution, the theorem is valid if only the mean and the variance of this common distribution exist. As mentioned in the paper, the theorem may be extended to cases of continuous and dependent variates. It is hoped that the above demonstration will serve the purpose of clarifying the basic idea of the central limit theorem. It is not practical, however, to prove the theorem, because it requires rather advanced mathematical techniques which are beyond the scope of the present discussion.[6]

Characteristic Values and Their Relations

The relation between C_s and C_v shown by Messrs. Alexander and Karoly

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6. For the interest of mathematically minded readers, it is noted that a complate proof of the theorem under general conditions was first given by A. Liapounoff.(81)(82) The proof for simple cases may be found in the book by Cramér.(1)

may also be obtained as follows: If $C_g = \overline{x} / M$, then Eq. 21 gives

$$C_g = e^{\sigma_y^2/2}$$

or

$$e^{\sigma_y^2} = C_g^2$$

Substituting this expression in Eq. 14,

$$C_g^2 = 1 + C_v^2$$

which is the relation derived by Messrs. Alexander and Karoly. Furthermore, if $\sigma_g = x' / M$, other useful relations may also be derived from Eqs. 21, 25, and 14. They are

$$\sigma_g = e^{\sigma_y}$$

or

$$\sigma_y = \log_e \sigma_g$$

and

$$C_g = \left\{\exp\left[(\log_e \sigma_g)^2\right]\right\}^{0.5}$$

or

$$\sigma_y = \left[\log_e C_g^2\right]^{0.5}$$

These relations will help the computation of σ_y from values of C_g or σ_g which are obtained from commercial log-probability paper, and eventually enables the determination of C_v and C_s.

Mr. Beard advocates the use of the moments of logarithms instead of the moments of the original variate for computing characteristic values. In other words, he would compute, from common logarithms of the variate, the characteristic values corresponding to $\overline{y}$, σ_y, and C_{s-y} in stead of $\overline{x}$, C_v, and C_s. Here, the notation C_{s-y} designates the coefficient of skew for y or for $\log_e x$. In this case, the table of K values can still be applicable since the coefficients can be easily transformed by means of Eqs. 11, 14, and 16. Taking Mr. Beard's example(77) for illustration, the annual flows during 1927 to 1946 in Willamette River at Albany, Oregon, are shown in Table D-2. The values of the mean, standard deviation, and coefficient of skew of the common logarithms of annual flows as computed by Mr. Beard are respectively: M = 4.944, $\overline{S}$ = 0.205, and Sk = 0.287. Converting these values to coefficients based on natural logarithms, the following are obtained:

$$\overline{y} = M \log_e 10 = 4.944 \times 2.3026 = 11.40$$

$$\sigma_y = \overline{S} \log_e 10 = 0.205 \times 2.3026 = 0.472$$

$$C_{s-y} = Sk\sqrt{(n-1)/n} = 0.287 \times \sqrt{19/20} = 0.280$$

Theoretically, the value of C_{s-y} should be zero if the distribution of the logarithms is strictly normal. The characteristic values using the moments of the original variate are transformed as follows:

By Eq. 11,
$$\bar{x} = \exp(\bar{y} + \sigma_y^2/2)$$
$$= \exp\left[11.4 + (0.472)^2/2\right]$$
$$= 99{,}840$$

By Eq. 14,
$$C_v = \left[\exp\sigma_y^2 - 1\right]^{0.5}$$
$$= \left[\exp(0.472)^2 - 1\right]^{0.5}$$
$$= 0.500$$

By Eq. 16,
$$C_s = 3\,C_v + C_v^3$$
$$= 3\,(0.500) + (0.500)^3$$
$$= 1.623$$

In order to check these transformed values against the actually computed values, the latter are computed as follows:

$$\Sigma x = 1{,}962{,}600 \quad \text{and} \quad \bar{x} = 98{,}130$$
$$\Sigma x^2 = 238{,}677{,}480{,}000 \quad \text{and} \quad \overline{x^2} = 11{,}933{,}874{,}000$$
$$\Sigma x^3 = 34{,}739{,}247{,}366{,}000{,}000 \quad \text{and} \quad \overline{x^3} = 1{,}736{,}962{,}368{,}300{,}000$$

By Eq. 38, $C_v = 0.502$

By Eq. 39, $C_s = 1.004$

It is interesting to note that the transformed values of $\bar{x}$ and C_v agree excellently with the corresponding actually computed values: $\bar{x} = 99{,}840$ vs. 98,130 with a discrepancy of 1.7% and $C_v = 0.500$ vs. 0.502 with a discrepancy of 0.4%. However, the values of the coefficient of skew do not agree, either using the moments of logarithms ($C_{s-y} = 0.280$ vs. 0) or using the moments of the variate ($C_s = 1.623$ vs. 1.004). Moreover, as the coefficient of skew for a short period of record of data is generally not reliable, a graphical adjustment of the value is therefore recommended.

The Frequency Factor

The writer appreciates the suggestion by Prof. Iwai and Messrs. Raff[2] and Lee are simplified solutions of Eq. 34. Mr. Raff's comment is:

"In looking at the complicated right-hand side of Eq. 34, it occurred to me that it might have been simpler to find the value of C_v corresponding to the given C_s by trial and error using Eq. 16, after which σ_y could be obtained easily from Eq. 14. But that is water over the dam."

In fact, the writer(79) has computed the value of σ_y in a way similar to the procedures suggested by the discussers, but he failed to mention it in the paper. He found the value of $e^{\sigma_y^2}$ corresponding to the given C_s by trial and error using Eq. 33, then obtained C_v easily from Eq. 14; and σ_y is simply equal to $\left[\log_e(e^{\sigma_y^2})\right]^{0.5}$.

Mr. Foster demonstrates the method of computing the probability of the mean value and lists values of I_v and the corresponding mean probabilities. It is noted that the method is also described by the writer in the paragraph immediately above the heading "Relation with the Extreme-Value Law," which is stated as:

"The probabilities at mean are those which occur when the frequency factor is equal to zero, as indicated by Eq. 28. From Eq. 29, with K = O, it is found that $K_y = \frac{1}{2}\sigma_y$, and the corresponding probabilities at mean were found from a normal probability function table."

Values of mean probabilities for $\bar{x}$ are listed in Table 2 in the paper for corresponding values of C_v. The corresponding values of I_v may be obtained by Eqs. 14 and 22b.

Corrections for Computed Characteristic Values

Several discussers suggest the methods for the correction of computed characteristic values. Prof. Gumbel and Messrs. Alexander and Karoly mention Fisher's extension of Bessel's correction factor, or $n^2/(n-1)(n-2)$ for bringing Eq. 39 to its theoretically more advantageous form, or

$$C_s = \frac{n^2}{(n-1)(n-2)} \frac{\overline{x^3} - 3\bar{x}\,\overline{x^2} + 2\bar{x}^3}{\bar{x}^3 C_v^3}$$

In fact, the Bessel correction and Hazen's correction are correlated in the whole procedure of curve-fitting, and a change in Bessel's correction should effect a change in Hazen's correction and vice versa. Based on this reasoning, however, Messrs. Alexander and Karoly have shown that Hazen's correction is generally greater. As the empirical nature of Hazen's correction is not satisfied anyhow, a better method for correction is desired.

Prof. Iwai introduces a correction factor for the coefficient of variation similar in form to Hazen's correction factor for the coefficient of skew, but the value of either b / $\bar{x}$ or F' should be assumed. Mr. Lee suggests a similar method of correction, and he goes on further to describe the theoretical procedure for the development of the corrections of C_v and C_s. By this procedure, it is possible to determine the corrections if the size of the sample and a desired confidence probability are given; but it remains an uncertainty to assume an optimum confidence probability.

A practical procedure of determining the correction factors is developed

by Mr. Foster. He has constructed a set of curves of the correction factors obtained by a laborious procedure. Since these curves are derived from actual sample plotting, they should give more reliable and practical results than the original Hazen formula. It should be noted, however, that these curves are based on planned samples and hence it is expected to produce discrepancies when applied to a particular observed sample. Furthermore, when these correction curves are applied to other types of skew-probability curves, it is understood that better results would be obtained from similar curves developed from plotting the planned samples on the special skew-probability curve under consideration. It is interesting to see that the number of items has certain effect upon the coefficient of variation, although it is not as great as that upon the coefficient of skew.

It should be noted that Hazen has suggested two procedures of curve-fittings: the method of adjusted coefficient of skew, described on pp. 48-56 of his book,(39) and the method of graphic coefficient of skew, described on pp. 56-58 of the same book. He indicates that the second method is the safest one to follow, and that it furnishes the ultimate tests. In the second method no correction factor is recommended for use since the best value of C_s is to be obtained by the graphic procedure and the adjustment is made automatically. Therefore, the inclusion of an adjustment factor in Eq. 39 of the paper, as commented by Mr. Bertle, is unnecessary.

Hazen's Approach vs. the Theoretical Approach

Mr. Bertle explains very clearly the basic distinction between Hazen's approach and table and the writer's theoretical approach and table. Hazen's approach is based on 25-point samples, whereas the theoretical approach is based on the population. When the method of adjusted coefficient of skew, the first method mentioned in the preceding paragraph, is used, either of the two approaches can be used provided that a satisfactory adjustment factor corresponding to the approach in use is available and such factor is applied to adjusting the size of the sample. In Hazen's approach the size is 25, and in the theoretical approach the size is the population. The population is undoubtedly the best and logical reference with which a sample distribution is to be derived and upon which the adjustment is to be based. This practice is universally accepted by all statisticians. Moreover, since Mr. Foster has developed a set of adjustment curves for the theoretical approach which is far better than Hazen's correction, the reason to replace Hazen's approach and table by the theoretically exact approach and table is therefore very apparent.

From the above discussion it can be easily seen that the first example given by Mr. Bertle is to compare two statistics of different basis, one from the 25-point sample and the other from the population. The concluding statement of this example can therefore be easily misleading.

In the second comparison made by Mr. Bertle, the coefficient of skew used in the theoretical approach should not be adjusted by Hazen's correction which was designed primarily for the 25-point samples. This example has unusually high values of C_v and C_s which result in extremely high values of adjusted C_v and C_s that are beyond the limit of the tabulated theoretical frequency factors. However, the writer has found that extremely-skewed distribution can be better fitted by the log-probability law with a reduced variate. For this particular example, a reduced variate of an exponential form similar in principle to that suggested by Prof. Iwai may be used. Taking $C_v = 1.37$, Eq. 14

gives σ_y = 1.028. The probability at mean, designated by P_m, occurs when K = 0 or $K_y = 0.5\sigma_y = 0.514$. The normal probability function table gives P_m = 30.3%. With $\bar{x}$ = 1 and σ_y = 1.028, Eq. 21 gives the median M = 0.59. A straight line could be plotted on a log-probability paper by passing the line through two points: $x / \bar{x} = 1$, P = 30.3%, and $x / \bar{x} = 0.59$, P = 50%. Due to the extreme skewness of the data, this line is not a best fit. A further adjustment produces a value of M = 0.46 and results in the plotted line as shown in Fig. D-1. This adjustment is graphical and corresponds to the graphical adjustment by Hazen. The line is adjusted to agree with the observed data which are plotted by Eq. 42. Theoretically, a reduced variate $y = \log_e x^b$ where b = 1.47 is used in this procedure of adjustment.

Relation with the Extreme-Value Law

Messrs. Alexander and Karoly are correct in saying that "the log-normal and extreme-value distributions appear equally flexible." However, it should be recognized in the same time that "each law has its own limitations," as stated by the writer. The statement quoted by the discussers is used by the writer only to show the fact that the use of the frequency factors based on the log-probability law for a given C_v will produce a family of curves that provide more possibilities for fitting purposes. On the other hand, the writer states:

> "It means that for a given value of C_s = 1.139, the extreme-value law furnishes a large number of conditions to fit curves with straight lines; while the log-probability law offers only one possibility of straight line fitting for which C_v = 0.364."

These statements are used with the purpose of indicating the mutual advantages and disadvantages of the two cases; that is, the advantage of one is the disadvantage of the other, and vice versa. The statement quoted by the discussers may be made clearer by restating it as:

> "It means that for a given value of C_v = 0.364, the use of the frequency factors based on the log-probability law furnishes a great number of conditions to fit curves which may be either straight or concave upward of concave downward as it appears on log-probability paper."

Plotting Positions

Prof. Powell has prepared as interesting list of plotting positions computed by six different methods. The writer was aware of the manifold methods of computation of plotting positions when he made a probability study of hydrologic events in 1950(83) in which he enumerated the California method,(84) Hazen's formula,(39) Bednarski's formula,(85) Brandt's formula,(86) Gumbel's formula(87) and method,(66) Kimball's formula,(88) Beard's method,(89) and the Thomas' table.(90) The California method gives the Horton recurrence interval in Prof. Powell's table. In making a frequency study of 35-year record of rainfall intensities at Chicago, Illinois, in 1948, the writer has constructed curves as shown in Fig. D-2. These curves indicate plotting positions which are expressed in terms of recurrence interval and computed by six different methods, except the Horton method which gives the exceedance interval. It should be noted in this plotting that the ordinate represents the order of items in descending magnitude and it indicates the discrete number. It is interesting to see from these curves that the California method, Gumbel's method, and the Kimball formula result in

plotting positions very close to one another, while the other methods produce a very wide range at high recurrence intervals. Prof. Powell's suggestion for including all the plotting positions that have been suggested would consider this wide range at high recurrence intervals, and it will be too uncertain to consider such wide range. It would be feasible, however, to first investigate all the available plotting positions, reject those which are shown to be unsatisfactory by experience or theory, and then use the remaining ones in Prof. Powell's proposal. While Mr. Bertle favors Hazen's formula and Prof. Gumbel advocates better ones, the pros and cons of certain methods that should be included for use in Prof. Powell's proposal will require a settlement by a comprehensive investigation. Such investigation is beyond the scope of this discussion. So far as the objection to Hazen's formula is concerned, a discussion by Thom(78) should be considered.

In connection with Eq. 42, it is not the writer's intention to claim the priority of this formula, and he should have used a more pertinent word "advocates" rather than the word "suggests." Within the writer's knowledge, the U. S. Geological Survey(91) has put this formula in actual use early in 1947 immediately after the Kimball proposal in 1946.(88) However, it is believed that the writer has first performed the mathematical proof of this formula by means of the theory of the mean number of exceedances.(55) After the publication of this proof in July 1953, Prof. Gumbel has shown the writer, in a letter dated October 8, 1953, a simplified proof which was prepared for publication in the Annals of Mathematical Statistics.

Extended Applications

The writer is glad to see many extended applications of the log-probability law as brought up by the discussers. These extended applications are as follows:

1) Prof. Iwai extended the log-probability law to bounded distributions by introducing the upper and lower limits of the variate. This is a treatment with one of the cases also presented by Slade.(92) Prof. Iwai should be complimented for his able demonstration of how the principle developed in the paper can be readily extended to and correlated with the problem of the bounded distribution. Similar to this treatment, the writer is studying the possibility of using a reduced variate in the form of $y = \log_e ax$ or $y = \log_e x^b$, where a and b are parameters. This treatment will make possible to extend the log-probability law to an infinite cases of straight-line fitting on the log-probability paper.

2) Prof. Iwai also mentioned about the "log-extreme value law," proposed by Kadoya almost simultaneously with Gumbel's new proposal.(93) The writer has the knowledge that the Illinois State Water Survey has used successfully the log-extreme value probability plotting for the studies of the drought of the State of Illinois and for the corresponding storage reservoir design. The drought defined in this study, however, takes into account the element of duration that the drought lasts rather than the annual minima of the discharge. This definition is more logical and desirable as far as the volumetric design of the storage reservoir is concerned. The logarithmic extremal probability paper is also used by Freudenthal and Gumbel(94) for tracing the number of cycles to fracture at different stress-amplititude treatments of nickel.

3) An extremely interesting extended application of the log-probability law is given by Messrs. Alexander and Karoly. The discussers prove

mathematically that all the moment distributions are log-normal with the same standard deviation of the logarithmic variate. They also show a very simple graphical method of using parallel lines to extend the application. For the first moment, the distribution is that of the volume of flow. This application should be valuable to drought and storage reservoir studies, in which the time factor or the period during which a certain volume or quantity is maintained is an essential factor in the design. For higher moments, the application to the size, surface, and weight studies of particles is particularly interesting and it shows a very promising future.

There is another interesting extended application of the log-probability law which is for the so-called "reversed log-normal" distribution. The log-normal distribution is a positively skewed, leptokurtic distribution. That is, the distribution is skewed to the right as caused by the extremes in the higher values distorting the distribution curve towards the right, and it has a higher degree of kurtosis (peakedness) than the normal curve. If the distribution is leptokurtic but negatively skewed, it can be fitted with the log-normal distribution curve by imagining it rotated about the mean so that the long tail points in the opposite direction, or towards left. Reversing the two sides of the log-normal distribution curve will result in the reversed log-normal distribution. In practice, the reversed log-normal distribution is fitted in the similar way as the log-normal, the reversal being effected simply by reversing the signs of C_v and C_s in the equations presented in the paper, for both C_v and C_s are negative in the reversed distribution. The reversed log-normal distribution is rare in hydrologic data, but it may be found applicable to certain engineering or scientific data which possess a leptokuric, negatively skewed distribution, such as the barometric pressure.

Curve-Fitting

It is true that the flexibility of curve fitting can be improved by increasing the number of parameters. However, too many parameters require more mathematical effort to determine them and eventually will render the procedure of fitting impracticable. It is writer's opinion that the number of parameters used should depend on the nature of the data and the precision of the result desired. For those hydrologic data which have a relatively short length of record and uncertain reliability of the quality, it seems that using more than two parameters is unnecessary.

The preference by Mr. Sammons to the use of the theory of least squares in curve fitting is welcome in practical applications as the writer has used the method elsewhere.[55] For establishing the confidence bands, the writer recommends the Gumbel method of the most probable value for which the writer has developed formulas and curves for practical applications.[55, pp. 29-31]

Several discussers have questioned the reason of replotting the curve on a special grid. It is to be noted that this step is suggested only for the sake of offering a graphical comparison with the observed data. However, it is optional as far as the acquisition of the results of the analysis is concerned.

Verification

The writer is happy to see many verifications brought out by the discussers in support of the theory developed in the paper.

Mr. Foster has accomplished a remarkable empirical verification of the writer's formulas as he is able to demonstrate that the results obtained by his laborious arithmetic integration agree closely with those computed by the formulas developed in the paper. For large values of I_V, discrepancies are expected unless the extreme values of x can be well controlled in the arithmetic procedure.

Prof. Powell's remark on the close agreement between Prof. Howland's observation on critical value of C_V and the writer's theoretical prediction of the value is extremely interesting. It adds much weight to the soundness of the mathematical treatment presented in this paper.

The rainfall studies made by Messrs. McIllwraith and Sammons are additional examples justifying the several of the many engineering applications of the log-probability law. The five sound statistical tests performed by Mr. McIllwraith are particularly invaluable.

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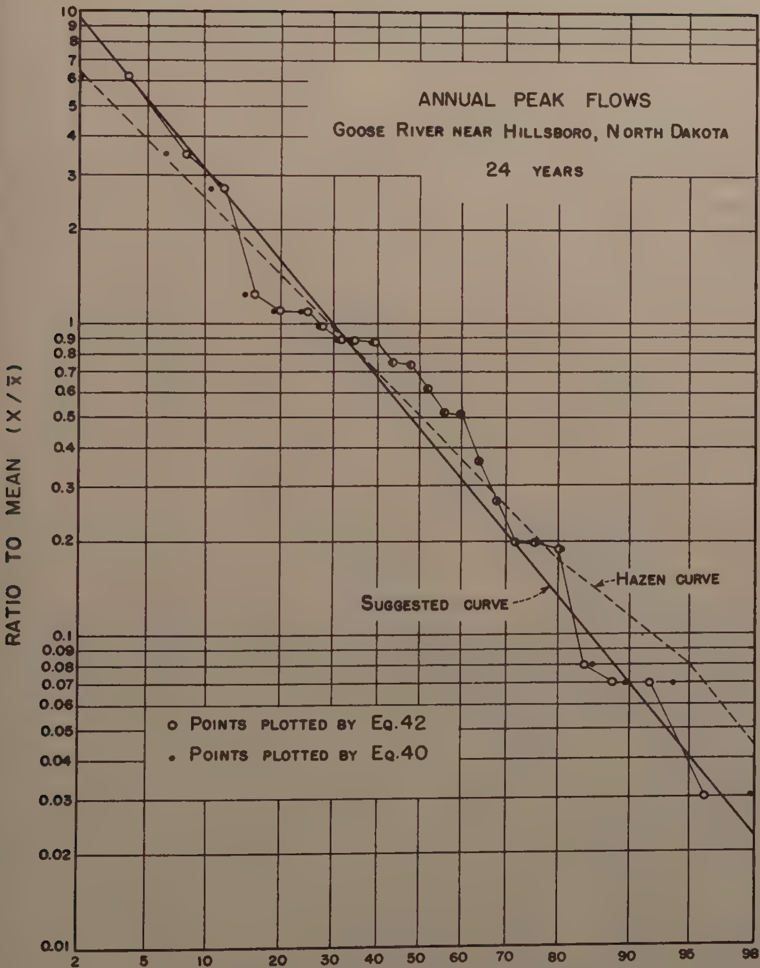

FIG D-1 PROBABILITY IN PERCENTAGE - EQUAL TO OR GREATER THAN THE GIVEN VARIATE

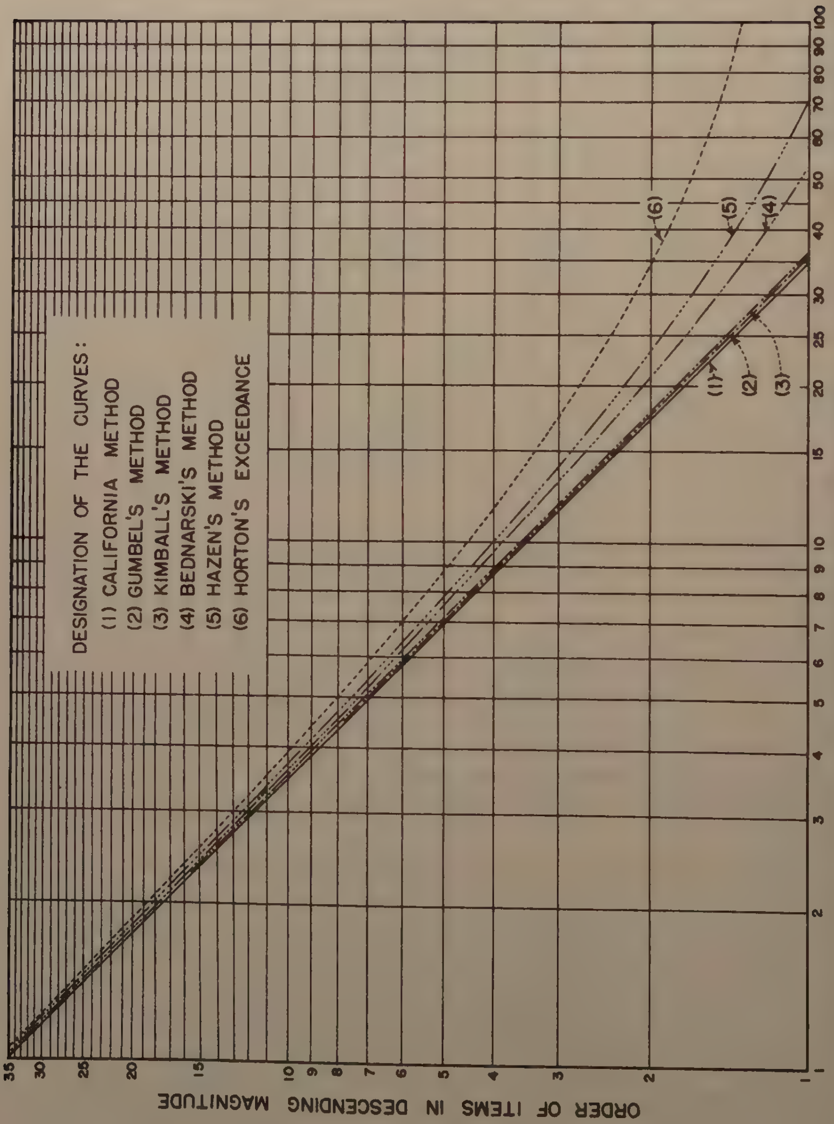
DESIGNATION OF THE CURVES:
(1) CALIFORNIA METHOD
(2) GUMBEL'S METHOD
(3) KIMBALL'S METHOD
(4) BEDNARSKI'S METHOD
(5) HAZEN'S METHOD
(6) HORTON'S EXCEEDANCE
ORDER OF ITEMS IN DESCENDING MAGNITUDE
RECURRENCE INTERVAL IN YEARS

FIG D-2

Table D-1 Demonstration of the Central Limit Theorem

Element No.	Population No. 1	2	3	...	r	Sum
1	${}_1X_1$	${}_1X_2$	${}_1X_3$	...	${}_1X_r$	${}_1X = \sum_{r=1}^{r=r} {}_1X_r$
2	${}_2X_1$	${}_2X_2$	${}_2X_3$	...	${}_2X_r$	${}_2X = \sum_{r=1}^{r=r} {}_2X_r$
3	${}_3X_1$	${}_3X_2$	${}_3X_3$	...	${}_3X_r$	${}_3X = \sum_{r=1}^{r=r} {}_3X_r$
.	.	.	.	...	.	...
.	.	.	.	...	.	...
.	.	.	.	...	.	...
Mean	$\bar{X}_1$	$\bar{X}_2$	$\bar{X}_3$	...	$\bar{X}_r$	$\bar{X} = \sum_{r=1}^{r=r} \bar{X}_r$
Variance	σ_1^2	σ_2^2	σ_3^2	...	σ_r^2	$\sigma^2 = \sum_{r=1}^{r=r} \sigma_r^2$

Table D-2 Annual Flows of Willamette River at Albany, Oregon

Water Year	Maximum Annual Flows	
	x in cfs.	$\log_{10} x$
1927	191,000	5.281
1928	74,000	4.869
1929	50,800	4.706
1930	84,100	4.925
1931	108,000	5.033
1932	134,000	5.127
1933	90,000	4.954
1934	70,200	4.846
1935	72,600	4.861
1936	120,000	5.079
1937	124,000	5.093
1938	92,600	4.967
1939	58,900	4.770
1940	58,300	4.766
1941	40,100	4.603
1942	83,400	4.921
1943	210,000	5.322
1944	44,400	4.647
1945	71,200	4.852
1946	185,000	5.267
	$\bar{x} = 98,130$	$M = 4.944$
	$C_v = 0.502$	$\bar{S} = 0.205$
	$C_s = 1.004$	$Sk = 0.287$

Discussion of
"SCALE RELATIONS AMONG SAND-BED RIVERS INCLUDING MODELS"

by Thomas Blench
(Proc. Paper 667)

THOMAS BLENCH,[1] M. ASCE.—Dr. Einstein has twice misquoted the writer as saying that the Froude Number has "nothing to do with hydraulics." The writer actually said, correctly, ". . . . dimensional considerations, having nothing to do with hydraulics, show that the scale of the ratio of gravity to inertia is unity when the Froude Number scale is unity."

It would be easy, and literally correct, to deal with Dr. Einstein's further comments by pointing out that he has misunderstood the application of regime theory to rivers, so that his Figs. A, B are irrelevant and his misunderstandings due to blaming the theory for his misapplication. However, this might suggest, by implication, that Dr. Einstein's own method of attack on the general problem was defective, whereas the writer has repeatedly expressed the belief[10,11,12] that the regime theory attack which he prefers, and the attack through speculative but ingenious formulas applied to laboratory flume data, as developed by Dr. Einstein, are complementary, legitimate, and both in need of development till a common viewpoint is reached; his own steps towards this common viewpoint are discussed elsewhere,[11,13] and have led to the inclusion of laboratory flume data in the framework of regime theory, which can now solve problems of transport as well as of self-adjustment of channel dimensions.[12] Moreover, he is aware that regime theory, with applications to river hydraulics, takes about six months to impart with reasonable efficiency to graduates, so is no easier to appreciate fully than any other major subdivision of hydraulics. Therefore he will attempt to clear the misunderstandings of his paper in terms of his own understanding of the exceedingly valuable work on American rivers by Leopold and Maddocks,[4] and, more recently, Wolman.[14] These investigators have proceeded with scientific objectivity and appear to have no bias in favour of any theory; their geomorphological outlook may be refreshingly new to the engineering reader. The writer knows that, in explaining the work of others, he may misrepresent some points of detail unintentionally; however, the risk seems justified, and the reader is expected to form his own final judgment from the original writings.

Leopold and Maddocks studied, for certain American rivers, the functional relation of b, d, V and suspended load, L, to discharge Q, not with any definite theory in view, but, like Gilbert with his flume experiments[15]—Gilbert is an acknowledged founder of modern geomorphology[16]—and Lacey with his canal data,[17] to find what was happening; this, of course, is the classic scientific method of getting facts correlated before starting theories, as exemplified in light, heat, sound, electricity, etc. They recognised two major subdivisions of their problem:

1. Prof. of Civ. Eng., Univ. of Alberta, Edmonton, Canada.

i. What functional relations exist for a given cross-section?

ii. What functional relations exist among various cross-sections along a river, or "downstream" to use an abbreviation?

A river engineer might call problem i. the problem of rating curves, with interest in a b, Q and an L, Q relation added to the usual d, Q one. (This is the problem studied by Dr. Einstein in his quoted reference;[5] he had data from three sites on the Missouri, two on the Salinas, and one on each of five others.) Simplifying greatly for present purposes, the writer would say that the results indicated that the Manning formula is a fairly good guide to a rating curve if the range of discharge is not too large, and circumstances not too special; but the goodness is probably a consequence of some compensation of disturbing effects whose nature may yet be extracted from the data.

Problem ii. is that of equilibrium relations, as distinct from problem i. which is of fluctuations about equilibrium. The writer would use the word "regime" for "equilibrium." Geomorphologists, like canal engineers, seem to accept the idea of equilibrium naturally, because they live with it; but, of course, they are aware that nature upsets equilibrium at long intervals and man may do so at short intervals. Some Civil engineers find difficulty in convincing themselves, so the matter is brought to attention here. With the idea of equilibrium accepted, it seems logical to assume that, at some definite equilibrium discharge at any river section or short reach, certain dynamical laws of equilibrium will be operating alone and they will be simpler than those for conditions off equilibrium. Therefore, the simplest results will be obtained in investigating problem ii. if b, d, V, L are plotted against equilibrium Q for various sites along a river. But what is the equilibrium Q at a section? Leopold and Maddocks, using gauging sites, had duration curves available and decided that the equilibrium Q would probably be that which was exceeded a certain proportion of the total running time. To find what that proportion was, they did analyses in terms of various tentatively chosen proportions and judged the issue on the proportion that seemed to give the maximum consistence. In the end they appear to have concluded that as good consistence as could be obtained resulted from departing slightly from the frequency idea and using the long term arithmetic mean discharge. So ultimately, their analysis "along the river," or "downstream," consisted of relating b, d, V, L to arithmetic mean discharge at a number of different sites with different arithmetic means. Of course, to have any hope of obtaining consistence, they would have to choose sections of similar geometric type, and avoid having bed-material, bed-load, suspended load, and sides differing too much within any one set of sections. The former requirement was met automatically by using gauging sections, which are normally chosen for having depth moderately uniform across. The latter appears, from the context, to have been kept in view; however, although nature produces a surprising number of rivers showing marked uniformity over large distances and large ranges of equilibrium discharge, an observer has to sacrifice something of what he would like if he wishes a large range of data.

The b, d, V correlations obtained by Leopold and Maddocks are all plotted, so that the reader can form his own judgment. Fig. 1 is adapted from their Fig. 9, which shows only some of the results but was drawn to illustrate that, in streams "differing markedly in physiographic setting, ... the rate of increase in depth, width, and velocity with discharge downstream may be similar among rivers despite marked differences in the width to depth ratio at

any particular discharge." They included line (8) for Madras canals to show similarity with canal results. The writer has drawn lines of slope 1 upon 2, and 1 upon 3, to assist the eye in judging how closely the plotted lines agree with the slopes that regime theory would prophesy if uniformity had been obtained for water-sediment constitution and side nature amongst the sections of a set. The results cannot be held to prove regime or any other theory without a site study of possible causes of inconsistence, as for river,(6) and verification whether uniformity does exist at the sites where there is consistence of slope of lines. After that, one should still exercise caution, since compensating errors frequently produce results causing support of an inexact theory. However, a study of all results, plus those of Wolman, does indicate that the behaviour of their river reaches, with presumably fairly constant F_b and F_s, is remarkably like what regime theory equations would suggest.

Translating the work described above into regime language the writer would say that Leopold, Maddocks and Wolman have performed a regime analysis of river systems strictly comparable with the Lacey analysis of canal systems. They have replaced the practically constant discharge of canals by a dominant discharge selected on a basis of consistence, and they have been handicapped by unavoidable variations of bed and side material. However, as a consequence of having paid as much attention as physically possible to the reduction of disturbing factors, they have achieved results or remarkable consistence. These results are in good accord with the regime theory finding that breadth varies, inter alia, with the square root of discharge, and depth varies with the cube root; the velocity relation is deducible from the other two so needs no separate consideration.

Reverting now to Dr. Einstein's Figures A and B, criticism of them should be easier. Figure A is not for "cross-sections along the river" so is not related to either the "regime flow prediction" line marked on it, nor to Fig. 1, both of which concern "along the river," or "regime" relations. If the river sites were suitably chosen the curves might be expected to have a general slope of 4 upon 1, since Leopold and Maddocks found that b varied roughly as the fourth root of Q on an average at a section; but as this relation is possibly a statistical accident, we cannot be sure. Actually a 4 upon 1 line drawn on Fig. A does strike a rough average inclination of the rather wavy lines. Fig. B first requires some correction as the so-called regime and Manning lines have been drawn with their slopes inverted, i.e. V has been shown proportional to the square of d instead of to the square root in the one, and to the 3/2 power instead of the 2/3 in the other. (Perhaps, however, the coordinates have merely become mixed.) Having made the correction the regime line should be removed since it concerns "along the river," whereas the diagram concerns "at a section." The corrected Manning line still does not look a very good indication of slope of curves, but regime theory combined with knowledge of rivers indicate many circumstances that would prevent agreement. At low stage, where the bed is inactive, the Manning formula could apply. But it could be upset by (a) tortuosity varying with stage, (b) rate of change of flow, (c) shifting of river control with stage, (d) water becoming turbid. At a stage at which the bed becomes active the regime theory line might be expected to apply (as to slope) over a range of discharge which could be appreciable if bed-load charge were not large. Some of the previous items could upset this, plus the facts that, (e) the bed-factor just after the start of bed movement is less than just before, and the difference may be considerable for large materials, (f) the bed-factor (V^2/d) must increase

eventually when the bed becomes superactive—and this puts Manning's formula out of commission too since it alters n.

Three other major points raised by Dr. Einstein are not covered by the preceding discussion. The formula of Inglis that he quotes represents a mathematical speculation and is obviously wrong, since it states that when the charge X tends to zero so does the bed-factor and, therefore, V—i.e. no charge, no flow of water! The expert in dimensional analysis will notice that the term on the right contains six variables and only the fundamental quantities length and time occur, so three more variables have been lumped into it than should have been for complete generality. This common slip in mathematics does not invalidate Inglis' views on the fact that charge is a relevant factor in real happenings. The question about Meyer-Peter's model, whose details the writer does not know, is answered in the text, which is careful to explain that various conditions can be relaxed if too much is not wanted from a model. That on the U. S. Army Engineer's alleged rejection of the Froude law is obviously incorrectly stated. First, distortion does not involve rejection of the Froude law—only the rejection of it in terms of width in which direction gravity does not act, so that the law is irrelevant; the writer's paper uses the Froude law vertically, but not horizontally, and that explains why distortion is needed. Second, any model that runs like a river needs a velocity, a depth and gravity in its full physical specification; therefore, from standard dimensional analysis, some measureable consequence must be expressible in terms of a Froude number in terms of depth. If the Army engineers preferred to keep the number implicit there would be nothing to prevent them doing so, but it would be there just the same. The writer had quoted a reference[8] containing a plot of Army Engineers' tidal river model scales, showing how exactly the Froude number controls the ultimate results in allegedly successful models.

Attention is drawn to a printer's error in Table 1. The index of b in row S, column Tbd, should be 3/2.

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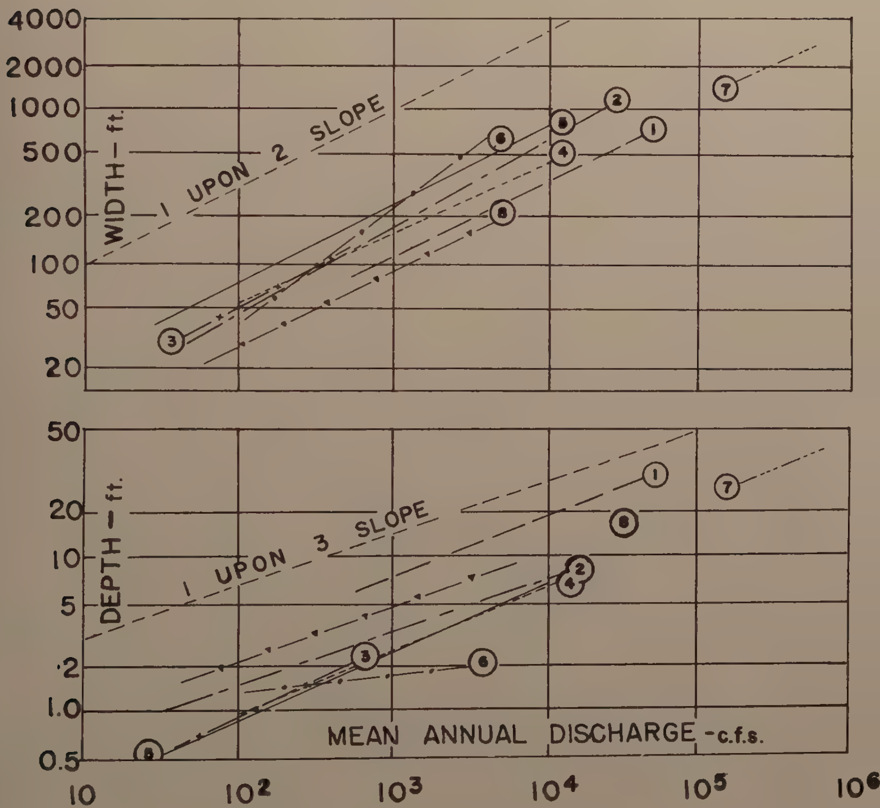

FIG. I. ADAPTED FROM LEOPOLD & MADDOCKS, FIG. 9 OF REF. 4.

Discussion of
"A MORE SIMPLIFIED VENTURI TUBE"

by J. C. Stevens
(Proc. Paper 678)

F. PADERI.[1]—The writer has read with the greatest interest the worthy paper in which he has found the confirmation of his own experimental determinations, done in the year 1936, and the concordance with what the writer himself deduced by the experiments in the publication[2] of year 1938, referring to the practical appliance of Venturi principle.

This publication has been largely reviewed with the reprint of figures in the review "L'ENERGIA ELETTRICA"—Fascicolo XI—Novembre 1938 pagg. 793-796—MILANO.

The experiments performed by the writer in the Hydraulic Laboratory of PISA University, concerned a Venturi tube type Herschel, with overall length of 2.024 m, made of three machined section: a 203 x 96 reducer of 539 mm length, a simiral diffusor of 1290 mm length and an intermediate throat section of 195 mm length.

In the Venturi tube were placed 113 piezometer taps concerning 38 different cross sections. For experimental purpose the piezometer taps were made each one of three small holes of 1.5 mm diameter at 2.5 mm intervals between them and drilled perpendicular to the inside wall of Venturi tube. The three small holes were placed in the way to concern the same inside cross section perpendicular to the axis of the Venturi tube.

Each ternary of holes opened into a tap of inside diameter of 11 mm. For a same cross section were made up to four taps at the ends of right-angle diameters.

To this tap, for the water charge reading, was connected a small rubber hose to the other end of which was connected a glass pipe (piezometer of 20 mm inside diameter vertically placed and opened to atmospheric pressure).

In the 1936 forequoted experiments the writer remarked a rippled trend of the piezometric line along the cylindrical throat.

Consequently the writer, in 1938 publication, pointed out explicitly the advantage to refer, as throat cross section, to a cross section placed upstream of the reducer end section for the regularity and steadiness of Venturi meter signaling.

It is to the writer source of satisfaction to notice the concordance in this regard of his and the Author's results.

1. Asst. Prof. of Hydraulics, Universita di Pisa, Italy.
2. "Determinazioni sperimentali sui misuratori Venturi," by Ferdinando PADERI, Arti Grafiche PACINI-MARIOTTI, Pisa, 1938, pp. 25, Paper No. 344 of VII serie of the publications by the Teaching Staff of the University Pisa, Engineering School.

Discussion of
"TIDAL CURRENTS AT INLETS IN THE UNITED STATES"

by J. N. Caldwell
(Proc. Paper 716)

W. DOUGLAS BAINES,[1] J.M. ASCE.—The author has provided practicing engineers with a simple classification system for tidal inlets based on the current behaviour at the mouth. There is a real need for this. Until the present every inlet has been considered as a separate item with no comparison made to other inlets. Four distinct cases were outlined in the paper but the author notes that many inlets are intermediate and do not fall clearly into one of the cases. He does not, however, indicate that cases 2 and 3 are particular examples of a general type of inlet. This is a large bay connected to the ocean by a channel which may be either much smaller than, or of the same size, as the cross section of the inlet. In the following paragraphs, the writer will outline the mathematical formulation of this general type and will discuss some of its properties. A complete treatment of the tidal mechanics is not required for this type; an extension of the simple concepts introduced by the author is all that is necessary.

Mathematical Solution of the Problem

Consider a bay with a surface area A and of sufficient depth that the time required for elementary waves to travel from end to end is smaller than the period of the tide. This assumption is analogous to that of the author that the length of the inlet be greater than one-quarter of the tidal wave length in the estuary. The discharge from the ocean into the bay will then be approximately the rate of rise in the bay water level multiplied by the surface area. Connecting the bay to the ocean assume that there is a uniform straight channel length L and cross section a, thus the mean velocity in the channel is

$$V = \frac{A}{a} \frac{dh_b}{dt} \tag{1}$$

where h_b = water level in the bay
t = time.

Integrating equation (1), produces the following expression for the tide level in the bay

$$h_b = \frac{a}{A} \int_0^t V \, dt \tag{2}$$

This is the continuity equation and does not define the flow completely. In addition, the dynamic equation for the channel can be framed which is a simplified version of that presented by Einstein.(1) It is assumed that the channel

1. Hydraulics Lab., Nat'l. Research Council, Ottawa, Ont., Canada.

is short enough that convective acceleration is negligible, in other words the constant discharge exists along the length of the channel at any instant. Equation (7) of Einstein,[1] thus reduces to

$$\frac{dV}{dt} = g\,\frac{h_a - h_b}{L} - \frac{gV|V|}{C^2R} \tag{3}$$

where
g = acceleration of gravity
h_a = tide level in the ocean
C = Chezy's resistance coefficient
R = hydraulic radius of the channel at mean tide.

For the tide in the ocean a simple cosine expression is the most convenient form to use

$$h_a = H \cos \frac{2\pi t}{T} \tag{4}$$

where
H = half range of tide
T = period of tide

Equation (3) is extremely difficult to solve because of the non-linear friction term. It can, however, be simplified by assuming that the absolute value of the velocity is constant throughout the tide cycle. Einstein[1] discusses the consequences of such an assumption and how it can be used in a practical calculation to give good results.

Combining equations (2), (3) and (4) yields the following basic equation:

$$\frac{dV}{dt} = \frac{gH}{L} \cos \frac{2\pi t}{T} - \frac{gV|V|}{C^2R} - \frac{ga}{LA} \int_0^t V\,dt \tag{5}$$

The solution of equation (5) is easily obtained by the Laplace transform method[2] with the condition imposed that it must have a definite period. Were there a transient effect, then the mean level in the bay would increase (or decrease) steadily. This is difficult to envision physically. It is found that the period of the current in the channel and the tide in the bay is identical to that in the ocean. Following are the exact expressions for these quantities:

$$V = -\frac{C_1}{C_3 - 1} \cos\alpha \sin\left(\frac{2\pi t}{T} - \alpha\right) \tag{6}$$

$$\frac{h_b}{H} = \frac{C_3 \cos\alpha}{C_3 - 1} \cos\left(\frac{2\pi t}{T} - \alpha\right) \tag{7}$$

in which the phase shift angle α is defined by

$$\tan\alpha = \frac{C_2}{C_3 - 1} \tag{8}$$

and the following constants are used for convenience

$$C_1 = \frac{TgH}{2\pi L} \qquad C_2 = \frac{g|V|T}{2\pi C^2 R} \qquad C_3 = \frac{gT^2 a}{4\pi^2 LA}$$

Properties of Solution

The biggest difference from the simplified solution given by the author is the phase shift defined by equation (8). It is the direct result of the inclusion of friction in the equation of motion. The tangent of the phase shift angle is proportional to the constant describing the frictional coefficients of the channel. It can be seen from equations (6) and (7) that the tide lags that of the ocean by α and V lags tide of the ocean by $\frac{\pi}{2} + \alpha$. In other words, the friction delays the tidal action by a certain amount and the current is normally $\pi/2$ out of phase with the impressed tide. This is in agreement with the author's equation for case 2, an estuary with an adequate inlet.

Another unusual property of the above solution is the attenuation factor shown by equation (7). This term, which is commonly used in electrical engineering, is here used to define the ratio of the tide range in the basin to that in the ocean. Two factors influence it, the first and most readily understood is friction. In equation (7) the term $\cos\alpha$ represents the effect of friction in reducing the tide range in the basin. The greater the friction the smaller will be the basin tide. The other factor is related to the resonance to the basin to the impressed tide wave. The term $C_3/(C_3 - 1)$ will always be greater than 1. In many cases C_3 is much larger than 1 and the entire term can be neglected. If $C_3 = 1$ then equations (6) and (7) produce infinite values. In this case the solution is no longer valid. As C_3 decreases in value (which is the same as the author's criterion as length of the estuary approaching one-quarter the tidal wave length) the tide in the basin increases in range thus the combination of friction and resonance effect can easily produce tides in the basin with a range greater than that in the ocean. Examples of this are shown in the author's Table 3, most of which are for estuaries whose lengths are of the proper order.

Application to the Paper

For the case 2 described by the author, friction is very small; therefore the phase shift should be very small and in many instances may be negligible. Assuming the phase shift to be zero, equation (6) reduces to that given by the author except for the effect of resonance. This effect also determines the attenuation factor as noted above. Not all of the inlets in Table 3 have a greater range in the basin because of this effect alone. If the width of the estuary narrows in the direction away from the ocean the tide range will increase on this account also. Thus more information is required before individual estuaries may be discussed in detail.

Applying the solution to the author's case 3, a basin with an inadequate entrance, requires more data to be supplied regarding the properties of the basin and entrance than is the case for the author's equation. Consequently, the two are not directly comparable. The solution given by equations (6) and (7) are more complete than the author's and provide additional details of the flow. One item which is in agreement with the author's statement is that for very large friction the phase shift may approach $\pi/2$. The current and tide are then in phase. For other cases of moderate phase shift the writer attempted to link the values in the author's Table 4 by means of the above solution. Consistent results were not found. It may be that the above solution is not a good approximation of the facts or it may be in the interpretation of the

information in the table. Further study on this problem should be made by someone familiar with the estuaries.

The value of the mathematical solution lies in its use in predicting changes in the flow regime of an estuary. Such problems as the enlargement of the channel, cut off of part of the bay, or the constriction of the channel, can be evaluated with a fairly simple set of equations. It is the hope of the writer that it will find some such usage.

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Discussion of
"SYNTHESIS OF RAINFALL INTENSITY-FREQUENCY REGIMES"

by David M. Hershfield, Leonard L. Weiss, and Walter T. Wilson, A. M. ASCE
(Proc. Paper 744)

WILLIAM H. SAMMONS,[1] J.M. ASCE.—This is another fine example of that which can be accomplished with multi-graphical correlation when applied to the much important subject of hydrology. The authors have briefly described the "good" points related to this subject. So that the reader may also appreciate the possible limitations, the writer would like to briefly discuss what he considers as the possible "pitfalls." The writer will also add a few suggested improvements.

The parameters will be discussed as they appear on the authors' Fig. 1.

1) Mean annual precipitation has been "normalized" by the cube-root transformation by Mr. C. K. Stidd.(1) What distribution was assumed to apply in Fig. 1? The return period of the mean annual precipitation varies with the type distribution assumed.

2) Mean annual number of thunder storm days have been studied by the writer and in general they were found to follow the Theory of Extreme Values (which is a skewed distribution). In this case the mean would have a return period of approximately 2-1/3 years. The scale used for the mean annual precipitation in Fig. 1 is the same as that used for the mean annual number of thunder storm days, although the unit length is greater in the latter case. This implies the same type distribution for both 1 and 2. Do they have the same distribution?

3) Mean annual days of rain equal to or greater than 0.01 inch scale is the same as that of 1 and 2 but the unit is smaller. This also implies that 1, 2, and 3 have the same distribution and the authors state that 3 "is inversely related to the average intensity during stormy periods."

4) Mean annual series of maximum daily precipitation amounts usually follow the Theory of Extreme Values although cases exist where the Log-Probability Law may result in a better fit. A different distribution would be the results for the latter.

5) 2-year, 1-hour precipitation (first estimate) appears to be a special scale.

6) 2-year, 1-hour precipitation (second estimate) similar to 5 is a special scale. 5 and 6 may or may not qualify as did 4 for the same distribution types.

The first estimate is the "true" measure of the correlation because of the deficiency of data needed under 4 in order to complete the second estimate. If the mean of the annual series of maximum daily precipitation amounts is known, a simple linear correlation will give an estimate of the 2-year, 1-hour precipitation.(2,3,4)

A combination of 1 and 2 was employed by W. D. Potter(5) in a correlation

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analysis. In this case Potter used the number of excessive storms instead of the mean annual number of thunder storm days although both may be used as an indices for a similar project.

The authors state that "The average length of record of the 134 U. S. First-Order Weather Bureau Stations used to define this relationship is 40 years." Does this mean that the base period of record was 40 years? Or does it mean that the arithmetic average of sample sizes (years of record) were 40 years? Supporting data is needed to show the periods of record, etc., so that the average period can be approximated.

The geographical distribution of stations should also be indicated. One of the most important criteria to be established before a correlation is to be conducted is the base period of record.(3,6,7) In Reference 6 and 7, it has been shown that the variation from the long term period of record can be very great unless the sample size is at least 25 or greater. Although the variation does not materially affect the 2-year return periods, the present practice of the U. S. Geological Survey is to graphically fit a curve about the mean (2.33-year return period). This would tend to minimize the variation that could exist for a 2-year 1-hour depth of rainfall. Correlation with such a small return period amount is like correlation between "dish water and soap." The 5-or 10-year 1-hour return period amount would have been a better base and probably open to less criticism than the 2-year return period, (additional remarks will be made in a later paragraph concerning the use of the 5-or 10-year 1-hour return period as a base).

The authors state that "To convert to the partial duration series which is based on all the high values of rainfall instead of only one per year, an empirically defined factor of 1.09 is included in the chart." The method(2) used to select "raw data" for the partial duration series is the weakest link in this conversion factor. It is a matter of opinion whether the final product would yield the same results. The writer prefers the extreme value theory where the return period is a quarter, half year, etc. to the partial duration series where an arbitrary definition may allow the use of events very closely related. This could easily effect the distribution of the raw data. The writer also prefers the use of regression equations instead of the empirical factors where the slope intercept is assumed to be zero although the range of the data does not say this even if there are physical limitations implied.

In the section on other duration frequencies the following comparison is made between the Kentucky study(3,4,8) and that of the authors:

Duration	Authors	Kentucky*
10 min.	3.0	2.9
15 min.	2.3	2.4
30 min.	1.5	1.6
60 min.	1.0	1.0
90 min.	0.75	-

*For the 2-year return period.

Very little difference, if any, in the average ratios to the 60-minute intensity exists.

Based on the authors "ratio of the 5-year and 10-year to the 2-year frequency," the following equations were developed by the writer and will be compared with that of the Kentucky study:

$$x = 0.84 + 0.40\,y \quad \ldots\ldots \quad (1\text{–W. B.})$$

$$x = 0.89 + 0.31\,y \quad \ldots\ldots \quad (1\text{–Ky.})$$

Where x is the ratio between amounts for specific return periods and y is the reduced variate defined as follows:

$$y = -\ln\left(-\ln \phi(x)\right)$$

where $T = \dfrac{1}{1 - \phi(x)}$

and $\phi(x) = \dfrac{T - 1}{T}$

where $\phi(x)$ is the probability equal to or greater than, and T is the return period of "frequency." In Reference 8, Appendix D, the writer has calculated an extensive table of values for selected return periods (a few copies of this Table 10 are available and can be obtained from the writer). Similar tables have been published elsewhere by the National Bureau of Standards.(4,9) Following are a few selected values.

$\phi(x)$	T	y
0.500	2	0.3665
0.800	5	1.4999
0.900	10	2.2504
0.960	25	3.1985
0.980	50	3.9019
0.990	100	4.6001

Only the first two decimal places are usually used in the above two equations. These two equations limit the user to the use of only the 2-year return periods as a basis of comparison. In order to eliminate such a situation, the following equations in terms of the reduced variate may be used to determine <u>any</u> ratio to a selected base return period. This equation is based on the Kentucky study(3,4,8) of 1-hour rainfall.

$$x = 1.1232 + 0.3963\,y \quad \ldots\ldots \quad (2\text{–Ky.})$$

where x is a network average 1-hour rainfall amount for selected values of the return period as may be represented by the y value. The ratios to the amounts may be developed from equation (2–Ky) since the writer has verified that the so-called "frequency factors" are independent of the durations from which they may be calculated. From eq. 2-Ky. the ratio between amounts of the 100-year and 2-year return period is calculated to be 2.32. Similarly for

the Weather Bureau study and Kentucky study the ratios for selected return periods would be as follows:

T	W. B. "Ratio"	Ky. "Ratio"
2	0.99*	1.00
5	1.44	1.36
10	1.74	1.59
25	2.12	1.89
50	2.40	2.11
100	2.68	2.32

*The theoretical value should be 1.00 but since these are based on the 2-, 5-, and 10-year return periods "ratio" a slight discrepancy exists.

In closing, the writer would like to acknowledge the helpful assistance in the past which has been rendered by the authors in his studies of a similar nature. Cooperation and personal relationships should be encouraged among private and governmental research workers.

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Discussion of
"THE IMPORTANCE OF FLUVIAL MORPHOLOGY
IN HYDRAULIC ENGINEERING"

by E. W. Lane, M. ASCE
(Proc. Paper 745)

E. KUIPER.[1]—One of the important phases in the planning of hydraulic engineering works is the prediction of future stream behaviour in order to avoid or minimize possible interference with the proper functioning of the engineer's project. For this reason the writer believes that Mr. Lane has made a valuable contribution to the science of river engineering by presenting a clear summary of the principles of fluvial morphology.

Although qualitative insight into a river problem is a prerequisite for an adequate quantitative answer, it must be recognized that qualitative insight along can hardly form the basis of sound engineering judgment. It may be true that a dam will cause upstream river-bed aggradation, but such a statement will immediately invite practical questions as to the rate and extent of sedimentation at particular points of interest. In order to demonstrate the difficulties that may be encountered in attempting to solve sedimentation problems in quantitative terms, an example of Class 3 will be considered. Suppose a dam is built as shown on Figure 1-C. This will cause the upstream riverbed to rise slowly from the original equilibrium grade to the final equilibrium grade, as shown in the figure. Let it be assumed that the problem under consideration is to determine the rate of river bed aggradation at a point, say fifty miles upstream of the reservoir. First of all the investigator needs to know the normal sediment transport in the river. Since river bed aggradation is mainly caused by the deposition of bed material, he will be largely interested in the bed material transport, of which the bed load may be an important fraction. Unfortunately, there are no accurate methods available at the present to measure bed load in river channels or to compute it from other information. Consequently he will start out his computations with basic figures that may be up to several hundred per cent in error. However, the investigator goes determinedly ahead and with the aid of available formulas and relationships, also containing a degree of uncertainty, he tries to evaluate, probably with a step by step method, first the deposition in the river channel due to backwater of the reservoir, then the new backwater curve, then the consequent decrease in sediment transport due to the flattening of the slope, and so on. For the sake of simplicity, it has probably been assumed that the river has a steady discharge, that the reservoir has a steady surface elevation, that the hydraulic roughness of the channel does not change with a change in sediment transport, and that the width and meander pattern of the river channel remain the same. Of course, all these assumptions introduce new errors of an unknown degree. Nevertheless the computations are carried on and the river begins to aggrade admirably, at least on paper. However, there

1. Senior Hydr. Engr., P.F.R.A., Dept. of Agriculture, Canada.

comes a stage whereby the water surface of the river begins to rise over its former natural banks. In nature, this will first cause deposition of sediment on the river banks and afterwards a possible breakthrough of the river towards lower lying adjacent land; but how to evaluate this in the computational process? After a last desperate struggle the engineer will probably give up and decide that the problem is too complicated for a detailed analytical approach.

The intent of the above discussion is certainly not to minimize the value of Mr. Lane's paper. On the contrary, an understanding of the principles of fluvial morphology, so eloquently presented by the author is of foremost importance. However, the writer wants to emphasize that this understanding is only of practical value when it can be followed up by expressing it in quantitative terms. It would seem that in many cases a detailed analysis cannot be carried through to a final solution of the problem. In the writer's opinion, the logical course to follow in the near future would be to collect systematically a great amount of field data concerning the regime of different types of rivers and how they are affected by artificial interference. The analysis of this data, combined with an understanding of the principles that are involved, may enable the engineer to provide the most reliable answer under the given circumstances.

W. H. R. Nimmo,[2] M. ASCE.—Generally fragmentation and resorting results in a progressive decrease of particle size of bed material going downstream. If, in the ideal case of a mature river draining a large area of uniform geological formation, for example sandstone, the particle size decreases uniformly then, as shown by the writer in the discussion of another paper,(1) the slope of the bed will also change at a uniform rate so that, if the curvature of the earth is disregarded, the profile of the stream becomes a parabola. This is borne out by observation on those portions of actual streams where the geological conditions are sufficiently uniform.

If a Class 1 profile change is caused by the diversion of a considerable portion of the flow of a river; the reduction in volume downstream of the point of diversion—corresponding to C in Figure 1-a—will be accompanied by a decrease in velocity and power to transport large particles. The bed material passing C will therefore not only be reduced in quantity but will contain a lesser proportion of heavier particles.

At the stage represented by the line $C^{11}A$, the material reaching point A will consist mostly of fine particles. The profile of the section $C^{11}A$ will be more curved than before the diversion. The gradual rising of the bed to the new equilibrium position $C^{111}A$ will be accompanied by a readjustment of the particle size grading of the bed material. This could result in stratification of the bed, but it is likely that remixing will be caused by deep scour and refilling by floods. The increased curvature of the profile in the Stage $C^{11}A$ has been indicated in Figure 1-a by the Author who has clearly illustrated the morphological changes that may be expected in normal streams, notwithstanding considerable variation in the flow.

Extreme variation in the rate of flow may result in unusual conditions such as are found in the Channel Country of South West Queensland, Figure A.

The rivers, the Georgina, Diamantina and Cooper (Cooper's Creek) reach

2. Brisbane, Queensland, Australia.

base level in Lake Eyre which is approximately 25 feet below sea level. For 90 per cent of their length, the average slope does not exceed one foot per mile. At a distance of 550 miles from Lake Eyre, corresponding to the location of the 10 inch isohyet, all three rivers enter areas of flood plains generally bounded by the remnants of a layer of laterite which once covered the region, where their profiles are almost identical sloping at 10-1/2 inches per mile.

The Cooper is typical of all three streams but is the largest and has the greatest rainfall. It drains an area of 90,000 square miles in Queensland, of which the 60,000 sq. miles above Windorah has an annual rainfall of 17 inches. The head of its largest tributary, the Thompson, is more than 900 miles from Lake Eyre. The flow is little or nothing during the dry spring and early summer and this condition may persist for two or three years during droughts when the annual monsoonal rains fail. At the other extreme the greatest flood (1891) is estimated to have had a peak flow of some 400,000 second feet and a volume of 11 million acre feet at Windorah.

Viewed from the air the Cooper in flood above Windorah appears as a strip many miles wide of interlacing (braided) channels, none of which are large. For some 200 miles south of Windorah, the river traverses 3-1/2 million acres of typical channel Country, a fertile plain spreading out to a maximum width of 40 miles between the high banks which at places close in to only two or three miles apart. Immediately upstream of such constrictions there is usually a large permanent water hole which may be many miles long and 30 feet deep, though elsewhere the channels are very shallow. The plain is raised only three to seven feet above a complex maze of distributary channels which are little more than gutters and frequently do not return to the main channels.

During large floods the distributaries overflow and portion or the whole of the plain is inundated by a slow moving shallow sheet of water which may take several months to pass a given point. Figure B illustrates the passage of the flood of April, 1949, having an estimated peak flow of 83,000 second feet and a volume of 3,600,000 acre feet. Notwithstanding the large peak flow, the hydraulic radius is very small and this is nature's method of checking the velocity of a large flood which otherwise would scour out a deep channel which the average flow could not maintain. Regarded as a natural irrigation project, the efficiency of distribution is remarkable. Of the 850,000 acre feet of the 1946 flood, 76 per cent was beneficially utilised in an application of an average of 14 inches over some 400,000 acres. The growth of pasture following such a flood is phenomenal but unfortunately large floods are infrequent, the most probable annual flood having a volume of only 200,000 acre feet which is little more than sufficient to fill the main channels, and replenish the water holes.

Fig. B, showing the flood of 1949 in Cooper's Creek, is used by permission of the Royal Australian Air Force, and may not be reproduced.

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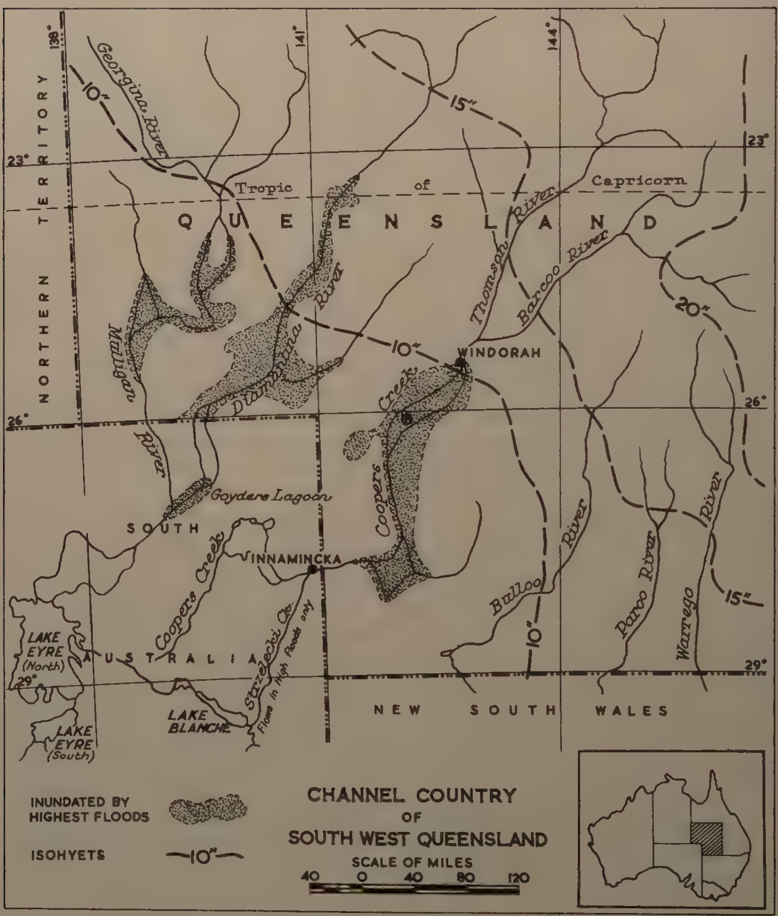

NORTHERN TERRITORY
QUEENSLAND
SOUTH AUSTRALIA
NEW SOUTH WALES
Tropic of Capricorn
138°
141°
144°
23°
26°
29°
Georgina River
Mulligan River
Diamantina River
Thomson River
Barcoo River
Coopers Creek
Goyders Lagoon
Strzelecki Cr.
Flows in High Floods only
Bulloo River
Paroo River
Warrego River
WINDORAH
INNAMINCKA
LAKE EYRE (North)
LAKE EYRE (South)
LAKE BLANCHE
10"
15"
20"
INUNDATED BY HIGHEST FLOODS
ISOHYETS
CHANNEL COUNTRY OF SOUTH WEST QUEENSLAND
SCALE OF MILES
40 0 40 80 120

Figure A.

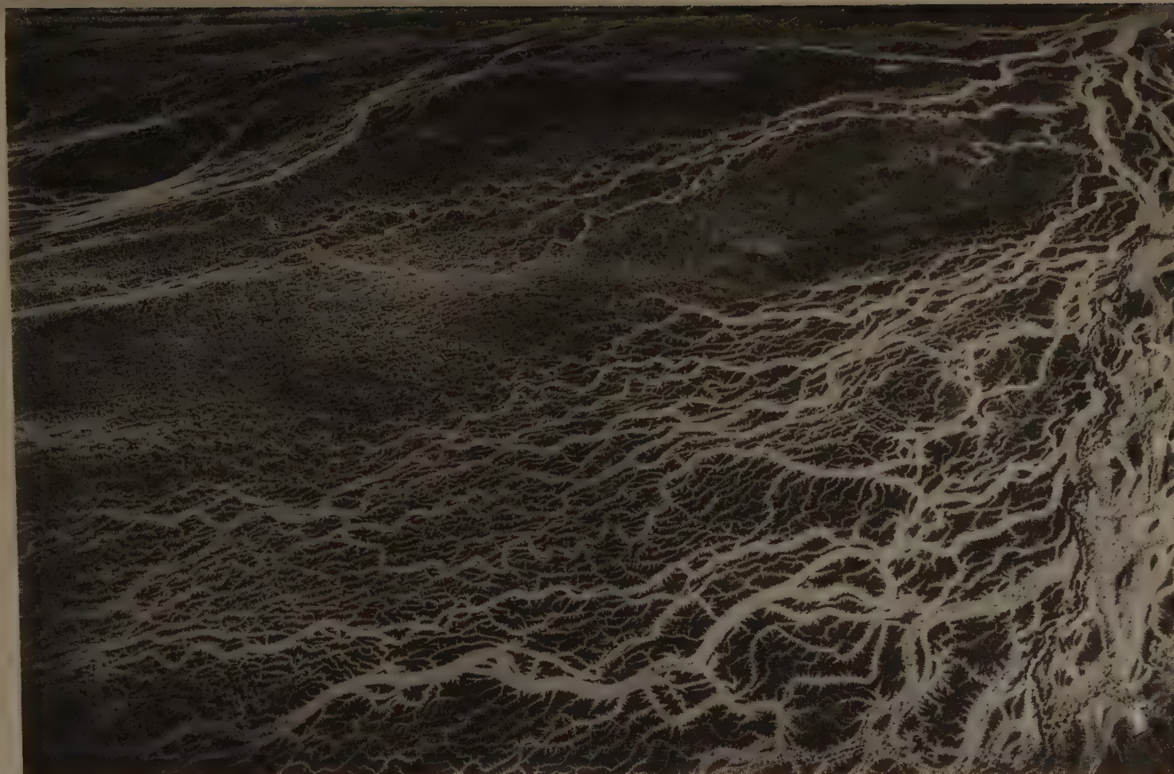


Fig. B. Oblique photograph looking north from 15,000 feet above the point marked by a circle in Fig. A. showing the flood of April, 1949, in Cooper's Creek downstream from Windorah. Scale approximately 1:30,000 at the lower edge of the photograph.

Discussion of
"MINIMUM PRESSURES IN RECTANGULAR BENDS"

by M. B. McPherson and H. S. Strausser
(Proc. Paper 747)

AHMED SHUKRY,[1] M. ASCE.—The fact that the flow conditions at a certain zone in a bend are similar to those in a free vortex motion has been evidenced experimentally in both closed and open bends. However, for the existence of this similarity the bend proportions should be within some geometrical limits. According to the free-vortex law, the velocity at any point in the section is inversely proportional to the radius. Consequently, as the inner radius r_i approaches the value zero, the velocity v_i of the inner filament approaches infinity and its pressure p_i/γ approaches a negative infinity, which is physically impossible. The writer raised this point in his discussion of the paper by I.M. Nelidov, M. ASCE.[5*] It was shown that the theoretical discharge computed according to the assumption of a free-vortex potential for any type of siphon spillways, was always nearly in agreement with the actual discharge, as long as the ratio r_o/r_i of the crown bend of the siphon was less than 3.o.

If the limiting inner radius of the bend is calculated on the basis of the available minimum negative pressure, which is about 24 feet of water, the limiting ratio $^{r_o}/_{r_i}$ will be so great that the theoretical result, obtained on the above basis may be questioned. Later experiments on open bends(1) showed that the validity of the free-vortex assumption is limited by the condition that the depth of flow at the inner side of the bend should not be less than the critical depth of the streaming state.

Obviously, there is also a certain minimum angle of deviation of the bend which limits the application of the theoretical free-vortex law. Experiments on open bends showed that when the deviation angle θ of the bend is less than 90°, the differences between the theoretical and the experimental values of the tangential velocities increase.(1) These differences, however, could be reduced by introducing in the free-vortex formula a coefficient U whose value was assumed to vary linearly between unity at θ = 90°, and zero at θ = 0 (straight conduit). Accordingly, when the deviation angle of the bend is less than 90°, the formula will be as follows:

$$v = U \times K/r$$

$$v = \left[\frac{r}{K} V + (1.0 - \frac{r}{K} V) \times \theta/90^\circ\right] \times \frac{K}{r} \quad \ldots\ldots (1)$$

1. Prof. of Irrigation, Faculty of Eng., Alexandria Univ., Alexandria, Egypt.
* The asterisk following a reference or figure number in this discussion means that the reference is to a footnote or figure of that number in the original paper.

The position of the section in the bend at which the maximum piezometric differential head (h_o - h_i) occurs, is not only a function of the bend angle, but also a function of both the Reynold's number R_n and the ratio $R/2c$ of the bend. Designating this section by angle θ_d, measured from the beginning of the bend, Fig. 1, the general relationship may be expressed as

$$\frac{\theta_d}{\theta} = C \phi \left(\theta/180°, R_n , R/2c \right) \ldots\ldots (2)$$

in which C is a constant and ϕ denotes a function. In the case of an open bend, the section at angle θ_d is characterized by the maximum surface depression at the inside wall and, as in a closed bend, by the most close similarity between the actual flow and the potential flow. The writer determined experimentally a range of relationship 2 for open bends, and the results are shown in Fig. 2. As can be noted, the ratio θ_d/θ decreases very rapidly as the parameter R/2c increases. On the other hand, θ_d/θ increases by increasing R_n. Since the characteristics of flow around closed and open bends are much the same, the writer suggests the use of the plots shown in Fig. 2, or their extrapolations, for determining the section of maximum piezometric differential head (h_o-h_i) in a closed bend.

The discrepancies in the total energy lines shown in Figs. 4* and 5* should not be attributed to the restoration of energy prior to the mid-sections of the bends as cited; for the total hydraulic energy once lost, cannot be regained. Accordingly, a correct total energy line should not rise above a preceding elevation. Referring to Fig. 1 and assuming a theoretical potential flow at the section of the bend whose angle is θ_d, the total energy will be constant from one filament to another through the section. Consequently, ($V^2/2g + h_{r_m}$) for the filament whose radius is r_m gives the ordinate of the total energy line H. However, the individual values of $V^2/2g$ and h_{r_m} do not represent the mean values of the kinetic energy and the potential energy, respectively, over the cross-section, and should always be referred to the filament whose radius is r_m. Furthermore, since the approximate potential flow condition is only established at the section of the bend whose angle is θ_d, the above procedure of locating the theoretical total energy line cannot be applied along the whole length of the bend.

Similar discrepancies of the total energy lines were also noted by investigators working on the analysis of the mean energy lines in siphon spillways, when piezometric measurements only were used in the analysis.[7*(2,3)] Such discrepancies were attributed to the non-uniformity of the velocity distribution across the different sections of the siphons. By integrating the energy curves, instead of the head curves, with respect to the areas of the examined cross-sections of siphon models, the writer could eliminate such apparent discrepancies in the mean total energy lines.[5*] The same procedure was also followed in open bends[(1)] and required, in both cases, velocity as well as pressure surveys over each cross-section. The general equations are as follows:

Mean velocity head $$\left(\frac{v}{2g}\right)_m = \frac{1}{VA} \int^{A} \frac{v^3}{2g}\, dA \ldots\ldots (3a)$$

Mean pressure head $\left(\frac{p}{\gamma}\right)_m = \frac{1}{VA}\int^A \frac{p}{\gamma}\, v\, d\, A$ ....(3b)

Mean position head $(y)_m = \frac{1}{VA}\int^A y\, v\, d\, A$ ....(3c)

Mean total head $(H)_m = \frac{1}{VA}\int^A \left(\frac{v^2}{2g} + \frac{p}{\gamma} + y\right) v\, d\, A$ ....(3d)

Accordingly, to obtain the correct mean velocity, pressure and position heads at any section from the normal head mean values $\frac{V^2}{2g}$, $\frac{P}{\gamma}$ and Y, respectively, correction factors α_v, α_p and α_y should be used. The values of these correction factors can be obtained according to the following equations:

the velocity head correction factor $\alpha_v = \frac{1}{V^3A}\int^A v^3 d\, A$ ....(4a)

the pressure head correction factor $\alpha_p = \frac{1}{VAP/\gamma}\int^A \frac{p}{\gamma}\, vdA$ ..(4b)

the position head correction factor $\alpha_y = \frac{1}{VAY}\int^A y\, v\, d\, A$ ..(4c)

the total head correction factor $\alpha_H = \frac{\left[\left(\frac{v^2}{2g}\right)_m + \left(\frac{p}{\gamma}\right)_m + (y)_m\right]}{\frac{V^2}{2g} + \frac{P}{\gamma} + Y}$ ..(4d)

in which,

$$V = \frac{1}{A}\int^A v\, d\, A, \quad \frac{P}{\gamma} = \frac{1}{A}\int^A \frac{p}{\gamma}\, dA, \text{ and } \quad Y = \frac{1}{A}\int^A y\, d\, A \quad ..(5)$$

Misleading results are sometimes obtained by correcting the velocity head $\frac{V^2}{2g}$ only, and neglecting the correction of the potential head $\left(\frac{P}{\gamma} + Y\right)$, using the eroneous assumption that the distribution of the potential head is similar to that in a parallel flow. Indeed, although the velocity head correction factor is not missed in current literature, no mention for the pressure and the position head correction factors has ever come to the writer's notice. For a theoretically potential flow in which the velocity and the pressure distribution functions are known, the magnitudes of the head correction factors α_v, α_p, α_y and α_H can be derived in terms of the dimensions of the conduit and the flow conditions. An example of such derivation is given in Appendix I, on the assumption of a free-vortex potential flow at a section across a bend, (Equations 6). In this case α_v and α_y are only dependent upon the bend geometry, as can be seen in Eqs. 6a and 6c, while α_p depends upon both the bend

geometry and the flow conditions in the bend, Eq. 6b. Fig. (3) shows experimental values obtained by the writer for α_v, α_p, α_y and α_H at the throat section of the summit bend of a siphon spillway. The pressures throughout the section were always negative and the datum for position and total heads was taken at the invert-level of the section. The differences between the experimental values and the theoretical values, as computed by Eqs. 6 can be considered as measures for the magnitudes of departure of the actual flow from the potential flow. For example, the mean experimental value of α_v is 1.331, Fig. (3), whereas the computed value according to Eq. 6a is 1.815; the ratio r_o/r_i of the bend being 4.88.

The writer agrees with the procedure explained in the paper for predicting, approximately, the critical cavitation conditions at the inside wall of the bend, provided that the total energy line at the entrance and the exit sections of the bend represent the true mean energy line. A modification which might decrease the differences between the predicted and the actual values of potential heads, especially in sharp bends, is to consider the initial drop of the total energy line due to the resistance of the bend. The bend-resistance factor ξ in the expression $\xi \frac{V^2}{2g}$ can be fairly estimated.(4) Published results of investigations on closed and open bends show that the energy lost due to the resistance of a bend is localized in two main drops, one at the beginning of the bend and the other in the straight tangent downstream its end.6* (1) Each drop is about 1/2 the total loss.

APPENDIX I. DERIVATION OF THE HEAD CORRECTION FACTORS ON THE ASSUMPTION OF POTENTIAL FLOW IN THE BEND

a) VELOCITY HEAD CORRECTION FACTOR α_v :

$$\alpha_v = \frac{\int^A v^3 \, dA}{V^3 A} = \frac{K^3 \int_{r_i}^{r_o} \frac{dr}{r^3}}{V^3 \, (r_o - r_i)} = \frac{K^3 (r_o^2 - r_i^2)}{2 \, V^3 (r_o - r_i) r_o^2 \, r_i^2}$$

$$= \frac{R \; r_m^3}{r_o^2 \; r_i^2} ,$$

substituting for r_m from Eq. 1, therefore,

$$\alpha_v = \frac{R}{r_o^2 \; r_i^2} \left(\frac{2c}{\ln r_o/r_i} \right) \quad \text{.....(6a)}$$

b) PRESSURE HEAD CORRECTION FACTOR α_p:

$$\alpha_p = \frac{\int^A p/\gamma \; v \; dA}{VAx1/A \int^A p/\gamma \; dA} = \frac{\int_{r_i}^{r_o} (H - v^2/2g - y) v \; dr}{V \int_{r_i}^{r_o} (H - \frac{v^2}{2g} - y) dr} ,$$

where y = z + r, Fig. 1. Therefore,

$$\alpha_p = \frac{1}{V}\left[\frac{\int_{r_i}^{r_o}\frac{HK}{r}\,dr - \frac{1}{2g}\int_{r_i}^{r_o}\frac{K^3}{r^3}\,dr - \int_{r_i}^{r_o}(z+r)\frac{K}{r}\,dr}{\int_{r_i}^{r_o}H\,dr - 1/2g\int_{r_i}^{r_o}\frac{k^2}{r^2}\,dr - \int_{r_i}^{r_o}(z+r)dr}\right]$$

$$= \frac{1}{V}\left[\frac{HK\ln {}^{r_o}/_{r_i} - \frac{K^3\,c\,R}{g\,r_o^2\,r_i^2} - Kz\ln {}^{r_o}/_{r_i} - 2cK}{2\,c\,H - \frac{K^2c}{g\,r_o r_i} - 2\,c\,z - 2\,c\,R}\right]$$

Substituting for V according to Eq. 1, $V = \frac{K}{r_m} = \frac{K\ln {}^{r_o/r_i}}{2\,c}$, and dividing all terms by 2 c, therefore,

$$\alpha_p = \frac{H - \frac{K^2\,c\,R}{g\ln r_o/r_i \cdot r_o^2\ r_i^2} - z - \frac{2\,c}{\ln r_o/r_i}}{H - \frac{K^2}{2g\,r_o\,r_i} - z - R}$$

$$\alpha_p = \frac{(H-z) - \frac{2c}{\ln r_o/r_i}\cdot\left(\frac{K^2R}{2g\ r_o^2\ r_i^2} + 1\right)}{(H-z) - \left(\frac{K^2}{2g\ r_o r_i} + R\right)} \qquad \text{.....(6b)}$$

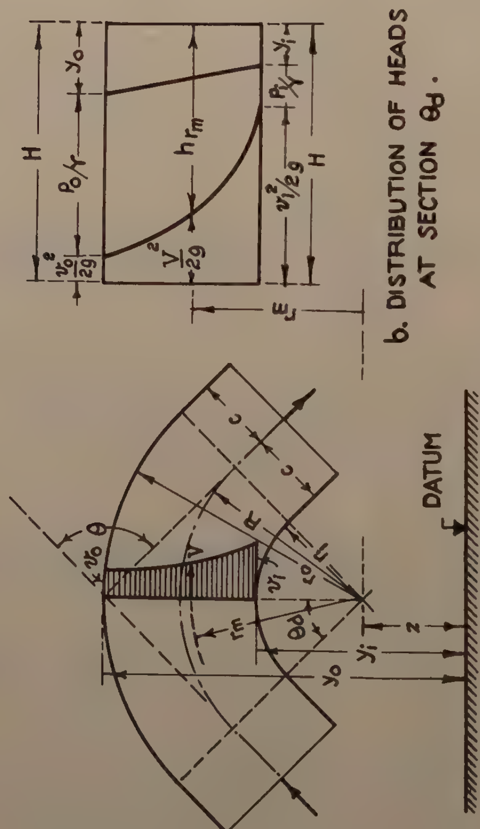
For calculating α_p, the magnitude of H can be estimated from the piezometric reading h at any filament of radius r, where,

$$H = h + \frac{K^2}{2g\ r^2} \quad \text{and} \quad K = Vr_m = \frac{Q}{b\times 2c}\times\frac{2\,c}{\ln r_o/r_i} = \frac{Q}{b\ln r_o/r_i}$$

c) POSITION HEAD CORRECTION FACTOR α_y :

$$\alpha_y = \frac{\int^A y\,v\,d\,A}{VA \times 1/A.\ \int^A y\,d\,A} = \frac{\int_{r_i}^{r_o}(z+r)\,\frac{K}{r}\,dr}{V\int_{r_i}^{r_o}(z+r)\,dr}$$

$$= \frac{2\,c}{K\ln r_o/r_i} \times \left[\frac{Kz\ln r_o/r_i + 2cK}{2\,c\,z + 2\,c\,R}\right] = \frac{z + \frac{2\,c}{\ln r_o/r_i}}{z + R} \qquad \text{.(6c)}$$



a. DISTRIBUTION OF TANGENTIAL VELOCITIES AT SECTION θ_d.

b. DISTRIBUTION OF HEADS AT SECTION θ_d.

FIGURE 1.

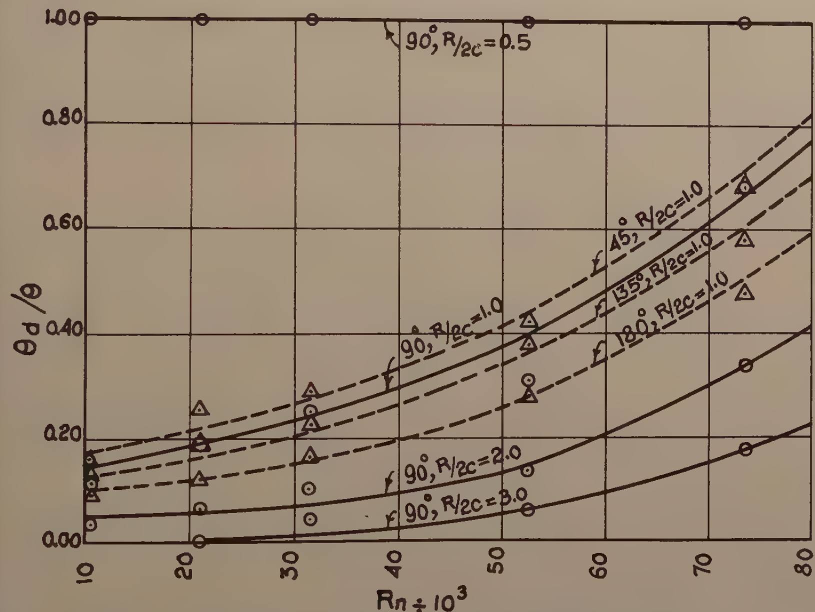


FIG. 2 VARIATION OF THE RATIO θ_d/θ WITH Rn.

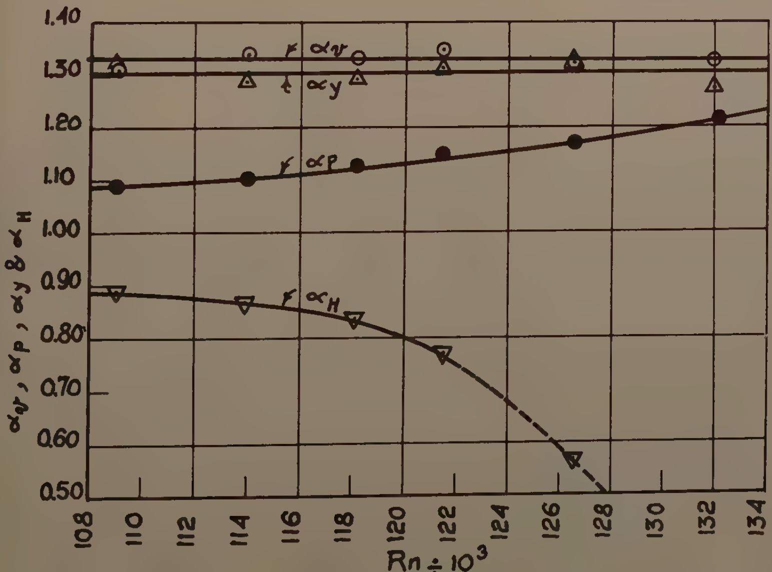


FIG. 3 VARIATION OF THE HEAD CORRECTION FACTORS WITH Rn.
N.B. The pressures are all negative, and the datum considered for the position heads y and the total heads H is at the invert-level of section.

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J. M. ROBERTSON,[1] M. ASCE.—Although agreeing with the authors as to the suitability of the method presented for determining the critical pressures liable to occur with the flow of fluids around bends, the writer disagrees with some of their interpretations and explanations. Extension of the analysis to other cases or more detailed description of the pressures around a bend may be less accurate if not even misleading. Thus, the basic theory used is independent of the angular position around the bend of the cross section under consideration, and yet Figures 4 and 5 show that changes occur as the flow proceeds around the bend. At 45° the solution is quite satisfactory, at other angles, both smaller and larger, it becomes progressively worse.

The application of potential theory to the analysis of flows in which viscosity plays an important part must be approached with extreme caution. Conduit flow (at Reynolds Numbers above 2500) is turbulent with strong velocity gradients near the walls. Interaction between this flow and the radial pressure gradients occurring in a bend produces a complex flow pattern. Even in circular bends secondary currents result which progressively change the flow pattern as the flow proceeds around the bend and downstream in the conduit. In straight conduits of rectangular section secondary currents are known to exist in association with the corners, so the bend flow is even more complex. Can one apply potential theory to such cases?

Potential flow theory is based on the assumption of irrotational motion. Turbulent flow fields are far from irrotational. In spite of this, in some areas of fluid mechanics we apply the potential flow solution to rotational flow fields as a superposition effect. The results appear satisfactory from a practical standpoint even if incorrect from a theoretical or mathematical one. An analysis of this situation, probably based on physical reasoning, is needed to indicate how far one can safely go in applying potential theory to rotational flows.

The writer questions whether the authors have presented a potential solution to the problem at hand. The relation utilized for the velocity distribution, $vn = K$, applies at some distance around a bend with an upstream uniform velocity distribution, a condition far from reality. Construction of the flow net, an elementary form of potential solution, indicates that the flow upstream from the bend is effected by the bend and that a fixed velocity profile will not occur for some distance around the bend. The velocity distribution relation used, and hence Equations 1, 2, etc., is not limited to angular position around the bend. Actually, this velocity law is that found for the so called free-vortex and will not obtain in a bend for quite a distance. Such a free-vortex

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flow pattern has been used as one basis for theoretical analysis of pump actions with some success.

The authors are probably cognizant of much of what has been stated above, but their paper is misleading in its implications. Thus the first conclusions concerning agreement between measured and theoretical pressures should be limited to a location 45° around the bend. Some warning of the complexities of the three-dimensional flow in bends is necessary for the benefit of those who may be interested in determining more information on the bend flow. Having measured the velocity distribution in the flow around circular bends [1,2] and in the pipe downstream, the writer has personally noted how the filament of maximum velocity wanders back and forth across the pipe. The situation in a rectangular conduit can hardly be any simpler.

In his studies of the flow around bends in circular pipe the writer compared the velocity distributions measured with the free-vortex law and found[2] that "the general trends predicted . . appear to be verified, but the actual distributions are modified greatly due to the effect of the velocity distribution in the approach pipe." In a rectangular conduit the aspect ratio of the conduit cross-section affects the flow distribution,[3] and conditions can hardly be closer to elementary theory than in a circular conduit. With only a modicum of agreement as to the velocity profile it is surprising that the integral of the velocity profile yields such a satisfactory relation for the maximum pressure difference. The details of the flow do not seem to matter.

Besides presenting the free-vortex flow analysis Lansford[10] presented a grosser analysis which he found more satisfactory in predicting the maximum differential piezometric head, represented by the bend co-efficient C_k. A transverse or radial element of fluid is assumed to go around the bend with velocity V and radius R. Application of the second law of motion to this element results in the expression,

$$C_k \doteq 4c/R = 4/x \tag{A}$$

This is much simpler than Eq. 2 and the writer has found it to agree well with test results from 45° taps on 4-inch[2] and 6-inch pipe of relatively sharp curvature ($x < 3$). For more gradual bends this relation is close to that of Eq. 2. Of the data presented by the authors, most of it is for bends of large curvature, but five points at $x = 2$ are presented in Figures 2 and 3—three of them agree with Eq. 2 and two with Eq. A. More data is needed before one can ascertain which is the more appropriate equation.

In discussion the experimental results presented, it is stated that no viscous effects were noted. No indication of the range of Reynolds numbers over which this applies is given. In most situations involving the measurement of pressure differences resulting from the flow (for example any type of differential head meter) the coefficient relating head and rate of flow is found to change considerably below some conduit Reynolds number, of the order of 10^5. For circular conduits the writer has evidence that this also occurs for the bend pressure difference as evidenced by the data shown in Figure A. In this data the reason for results from two of the bends being displaced from the rest may be piezometer errors or inaccurate determination of $X = 2R/D$. Presumably the authors' tests were run at higher Reynolds numbers where the bend and discharge coefficients are constant.

In Figure A for a Reynolds number of 10^4 the discharge coefficient is seen to be reduced by more than 20 per cent. This means the bend co-efficient and

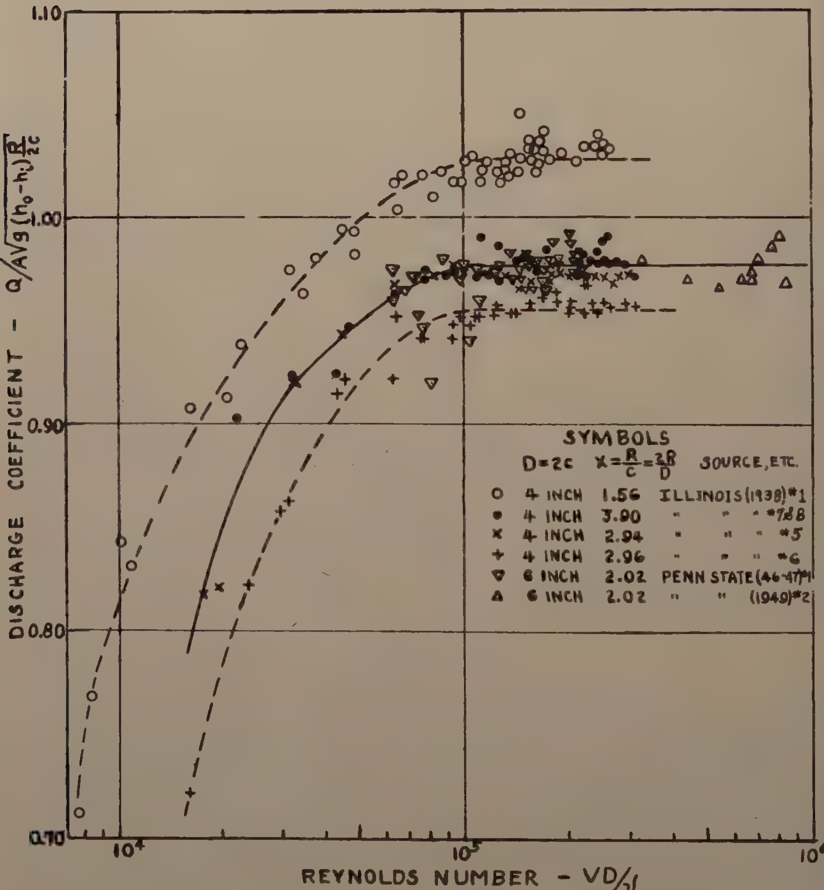

FIG. A VARIATION WITH REYNOLDS NUMBER OF HEAD DIFFERENTIAL AT 45 DEGREE TAPS IN CIRCULAR PIPE, AS EVIDENCED BY DISCHARGE COEFFICIENT

hence (h_o - h_i) is increased by nearly 70 per cent. There seems to be no reason why similar effects should not occur at Reynolds numbers below 10^5 for bends in rectangular conduits.

The analysis of the energy and pressure distributions around the bends as illustrated by Figures 4 and 5 in terms of the radius r_m is confusing. For, as noted in the footnote, r_m has no significance except at the cross-section of maximum (h_o - h_i). It is also stated that r_m must approach R towards the entrance and exit tangents of the bend. Actually, r_m has significance only by its definition in the expression for the free-vortex type velocity distribution $vn = Vr_m = K$. For the ideal (uniform) velocity profile at the entrance, r_m becomes indeterminate. For an actual flow condition V occurs as a contour in the cross-section and r_m is multivalued. The "mean" piezometric head is apparently the value found on the noncurved sides of the bends at a radius of r_m. Why should these pressures be associated with the mean velocity through a unique value of the Bernoulli constant? Secondary currents must modify the conditions greatly, particularly near the noncurved sides of the bends.

It is the writer's opinion that the "potential" flow relationships introduced in this paper are only of value in indicating what portion of the maximum pressure difference due to the bend should occur as a pressure reduction along the inside curve. Verification of the relation for this minimum in indicated for two bends of relative curvature x = 2.0 and 3.2. Additional verification, particularly at low values of x would appear to be desirable.

Since the writer agrees with the authors' method for predicting the pressure minimum in bends, and feels that a most interesting study has been presented in this paper, this discussion may appear unduly critical. There were two objectives to what has been presented. One was to amplify the picture presented of the flow conditions. The second was to question the validity of the theory as more than a first order indication of what occurs in the flow.

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3. "Modern Developments in Fluid Dynamics," edited by S. Goldstein, Oxford Press, Vol. 1, 1938, pp. 84-87.

E. F. RICE,[1] A.M. ASCE.—There is, apparently, no mathematical theory available to describe flow in a conduit bend except that used by the authors: that of 2-dimensional potential flow. Real fluids at high Reynolds numbers do follow the streamlines predicted on the basis of 2-dimensional potential motion, but only in regions near the upstream ends of disturbances to uniform flow. At a relatively short distance downstream, turbulent real fluids undergo adjustments to complex patterns not governed by these tenets.

In the case of bends in channels or pipes, spiral motion develops, which becomes noticeable somewhere between 45^o and 60^o downstream from the

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bend entrance. Farther downstream, the spiral motion becomes strongly developed, dying out only in the downstream tangent. The spiral can be shown to be the consequence of velocity distribution for potential flow, which has been observed to match theoretical expectations near the bend entrance. (1 p. 63)

The authors have shown that the maximum difference between internal fluid pressure at the inside and at the outside of a conduit bend occur on a radius about 45° downstream from the bend entrance, and that beyond this radius, the difference is sharply reduced, becoming zero somewhere in the straight pipe downstream from the bend. It is as if the mechanics of flow require that the fluid be turned through a deflection angle of about 45° before the flow conditions appropriate to the geometry of the bend can be established. Once established, the pattern rapidly breaks up, and the flow conditions downstream are sufficiently chaotic to prevent rigorous mathematical analysis at present.

The analysis presented by the authors should be considered valid especially for the region where the maximum pressure difference occurs, (the 45° station) and to a lesser extent for the region upstream from this. It is not valid in the region of disturbed flow downstream from the 45° station. Also, potential flow cannot be assumed for bends in series unless separated by sufficient straight sections of pipe to allow the flow to resume symmetrical velocity distribution upstream from each bend considered.[2]

It is to be emphasized, however, that the analysis presented by the authors is valid at the point of minimum pressure, and that this is the critical point for cavitation. Design of non-cavitating conduit bends may therefore be helped considerably by consideration of these principles.

The authors "have in mind the design of large bends which are frequently part of dam outlet structures, siphons, and other large conduit bends, Particularly those with limited or small 'back pressures'." Particularly in siphon bends, cavitation may become serious, since the upper bend is almost always above the elevation of the forebay surface. In addition, the siphon may be designed with an expanding lower leg (draft tube) in the interest of achieving the highest possible discharge coefficient. In such a case, the "back pressure" may be negative, and cavitation may be expected.

During a model study made by the writer in 1952 for a large siphon spillway (since built at a power development near Oregon City, Oregon) pressures were obtained as low as the vapor pressure of water. The cavitation taking place on the inside of the bend could easily be observed at the 45° station through the clear plastic walls of the model. In this case, cavitation in the model had been expected as a result of analyses of data obtained on another model. The authors' method would also have served to predict the existence of pressures low enough to induce cavitation. (1, p. 57, p. 107-8).

It may be that vanes or "splitters" can be used in bends in hydraulic passages as they often are in gas conduits (wind tunnels, for instance). It is possible to design a curved "splitter" which will divide a conduit bend of critical radius into two bends neither of which will be in danger of cavitation. An examination of the authors' equation (4) shows that the increase in the average value of x resulting from placing one or more "splitters" in the bend will greatly reduce the cavitation number.

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Discussion of
"FLOW GEOMETRY AT STRAIGHT DROP SPILLWAYS"

by Walter Rand
(Proc. Paper 791)

A. J. PETERKA[1] and J. N. BRADLEY,[2] M. ASCE.—Mr. Rand is to be congratulated for his excellent treatment and presentation of the dimensionless curves of Figure 4. The curves provide a simple means for determining the elements of straight drop spillways.

The writers, however, are concerned over the practice of using shallow flow depths in a model to predict the performance of large prototype structures. In Table 1, for example, when the drop number D is 0.00132, $\frac{d_1}{h} =$ 0.0339. For the model drop height of 7.79 inches, d_1 is therefore 0.26 inches. Moving vertically down the $\frac{d_1}{h}$ column, d_1 is found to be, successively, 0.41, 0.59, 0.77, 0.95, 1.12, 1.29, and 1.48 inches. Thin jets of 1/4, 1/2, or even 1 inch are subject to excessive friction losses when compared to the thicker jets of prototype structures and may influence the subsequent measurements of other depths or dimensions in the same system. However, since Mr. Rand has also used test points recalculated from Moore's data, in which d_1 was somewhat greater, and has plotted all the points to obtain a single curve, it may be possible that the dimensions of a structure obtained from Figure 4 will be adequate. To illustrate the effect that shallow d_1 values might have on other dimensions of the structure, the following discussion will be of interest.

Mr. Rand states that he made no measurements of the length of the hydraulic jump but used "$L = 6\ (d_2 - d_1)$, averaging the measurements of Bakhmeteff and Matzke,[7] and Moore."[5] In the paper by Bakhmeteff and Matzke the concluding paragraph states

> "The writers are fully conscious that the size of the flume and certain imperfections of its construction militate against a greater precision in the results."

In the discussion of this paper several of the discussers indicated that they had reason to believe that the jump was longer than shown, and that the narrow test flume used by Bakhmeteff and Matzke probably introduced side wall friction which reduced the length of the jump. Moore,[5] in his conclusions,

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2. Hydr. Engr., formerly U. S. Bureau of Reclamation, now Bureau of Public Roads, Washington, D. C.
7. The Hydraulic Jump in Terms of Dynamic Similarity by Boris Bakhmeteff and Arthur E. Matzke, Transactions, American Society of Civil Engineers, Volume 101 (1936), p. 630.

states that "The length of jump below a fall appears to be about 20 percent longer than the jump below a sluice." Experiments performed at the Federal Institute of Technology, Zurich, Switzerland, in a flume 0.6 of a meter wide show the jump to be some 20 percent longer than found by Bakhmeteff and Matzke. On the other hand, tests made at the University of Berlin in a flume 0.5 of a meter wide show the jump length to be somewhat shorter.

Because of conflicting data and opinions, the writers made a series of 120 tests in the Hydraulic Laboratory of the Bureau of Reclamation in Denver, Colorado, to determine the jump length, using test flumes 1.0, 1.5, 2.0, 4.0, and 4.9 feet wide. Discharges per foot of width varied from 0.6 to 7.1 cfs.

Several methods of determining the length of jump were attempted, but visual observations proved to be the most satisfactory. The length of jump was judged from a practical viewpoint; the end of the jump would represent the end of the concrete floor and side walls of a conventional stilling basin. In all cases the jump was considered to end at the point where either the bottom currents began to rise from the floor or where the water surface of the jump just reached tail water elevation, whichever was the longer. Since the end of a jump does not remain fixed, but pulsates up and downstream, it was necessary for the observers to visually average the pulsations, resulting in some variation in the jump lengths. With a little practice, however, it became possible to duplicate jump length determinations on identical runs to within about 5 percent.

The results from the five flumes were plotted in dimensionless coordinates, jump length L divided by conjugate depth d_2 versus the Froude number of the incoming flow, using d_1 and V_1 just upstream from the jump in the equation $F = \frac{V_1}{\sqrt{gd_1}}$. By producing jumps having identical Froude numbers for the incoming flow in several of the flumes it was possible to evaluate the effect of flume width, unit discharge, and d_1 on the jump length.

It was immediately apparent that when d_1 was less than about 0.10 foot in the 1.0-foot flume the jump lengths became too short with respect to the average curve. Jumps of the same Froude number produced in wider flumes and at great depths d_1 showed uniform results close to the average curve. No trend toward longer jumps was apparent when the 4.9-foot flume was used rather than the 2.0-foot flume. The relation between jump length and the Froude number determined by the tests is shown in Figure 1.

To provide a direct comparison with Mr. Rand's curve, $\frac{L}{h}$, in Figure 4, the writers have computed their jump lengths in terms of the author's method of presentation and tabulated them in Table 1. Columns 1 and 2 are the drop number and the Froude number, respectively. Column 3 gives the ratio of jump length to d_2 found by the writers for a particular Froude number. Column 4 gives the ratio $\frac{d_2}{h}$ taken from the curve of Figure 4. Column 5 is the product of Columns 3 and 4. Column 6 gives the ratios taken from the author's curve $\frac{L}{h}$ of Figure 4. Column 7 shows the greater jump length, in percent, found by the writers.

To illustrate in practical terms the difference in jump lengths shown in Column 7, assume a unit discharge, q = 200 cfs, in the following examples.

In the first case, assume V_1 = 80 ft/sec and d_1 = 2.5 ft. Then conjugate

Table 1

D (1)	F (2)	$\frac{L}{d_2}$ (3)	$\frac{d_2}{h}$ (4)	$\frac{L^*}{h}$ (5)	$\frac{L^{**}}{h}$ (6)	% diff (7)
0.001	9.02	6.1	0.14	0.85	0.78	+9.0
0.0010	6.55	6.1	0.26	1.59	1.40	+13.6
0.0100	4.78	6.0	0.48	2.88	2.40	+20.0
0.1000	3.47	5.5	0.90	4.95	4.10	+20.7
1.000	2.53	4.9	1.70	8.33	6.80	+22.5

*Peterka and Bradley.
**Rand.

$d_2 = 30$ ft and $V_2 = 6.5$ ft/sec. The Froude number of the incoming flow $F = \dfrac{V_1}{\sqrt{gd_1}} = 8.9$ which for practical purposes agrees with the values of line 1 of Table 1. The length of jump computed from Column 3 is $6.1 \times 30 = 183$ ft while the length given by $L = 6\ (d_2-d_1) = 168$ ft. The latter basin would therefore be 15 feet too short.

In the next case, assume that $V_1 = 52.6$ ft/sec and $d_1 = 3.8$ ft for the same unit discharge. The conjugate $d_2 = 24$ ft, $V_2 = 8.3$ ft/sec, and $F = 4.76$. This latter value corresponds to F in line 3 of Table 1. From Column 3 the jump length then would be $6.0 \times 24 = 144$ ft while from $L = 6\ (d_2-d_1)$ the length would be 120 ft. Thus, the latter basin would be 24 feet too short.

In the final example, assume that $V_1 = 33.3$ ft/sec and $d_1 = 6$ ft. Then $d_2 = 17$ ft, $V_2 = 12$ ft/sec, and $F = 2.40$. See bottom line of Table 1. The jump length from Column 3 would be $4.9 \times 17 = 83$ feet and from $L = 6\ (d_2-d_1)$ would be 68 feet, making the latter basin 15 feet too short.

The writers are convinced, therefore, that the jump length, from a practical viewpoint, is longer than shown by the author's curve in Figure 4. Figure 2 shows the writers' and the author's curve plotted for comparison. The writers believe that the differences in jump length may be explained as follows.

The experiments of Bakhmeteff and Matzke were performed in a flume 6 inches wide, with limited head. The depth of flow entering the jump was adjusted by a vertical slide gate. The maximum discharge was approximately 0.7 cfs and the thickness of the jet entering the jump was 0.25 foot for a Froude number of 1.94. The results up to a Froude number of 2.5 are in agreement with the present experiments. To increase the Froude number, it was necessary for Bakhmeteff and Matzke to decrease the gate opening. The extreme case was for a discharge of 0.14 cfs and d_1 of 0.032 foot, for $F = 8.9$ These are considerably smaller values of discharge and d_1 than were used in the writers' experiments. Thus, it may be reasoned that as the gate opening was decreased in the 6-inch-wide flume, frictional resistance in the channel downstream increased out of proportion to that which would have occurred in a larger flume or a prototype structure. Thus, the jump formed in a shorter length than it should. In laboratory language, this is known as scale effect

and is construed to mean that prototype action is not faithfully reproduced. The writers, therefore, believe that this was the case for the major portion of the jump length determinations by Bakhmeteff and Matzke.

The upper curves of Figures 1 and 2, therefore, provide information to determine the length of paved apron and stilling basin necessary to contain the entire pump. Use of these lengths will result in maximum energy dissipation on the apron, minimum wave heights downstream from the apron, and minimum scour at the end of the paved section. It is recognized, of course, that improved action, shorter apron lengths, or both, may result from prope placement of baffle piers or sills.

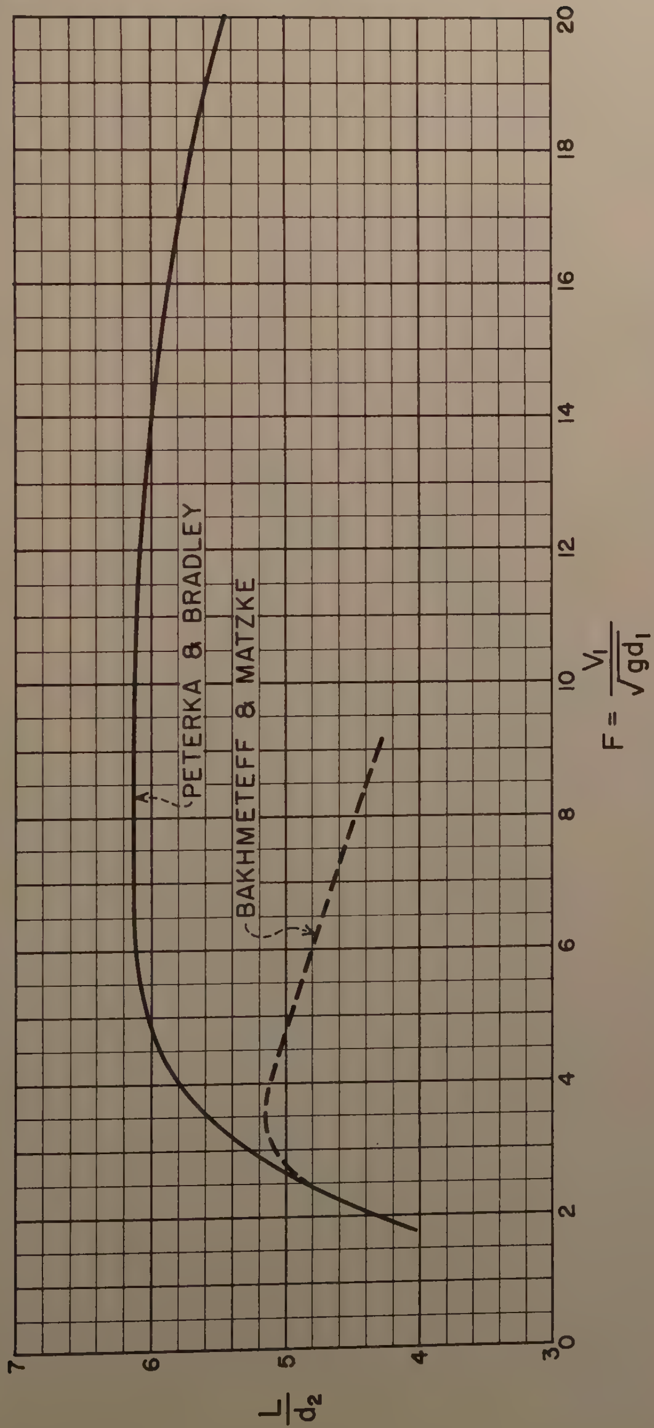

FIGURE 1. LENGTH OF HYDRAULIC JUMP ON HORIZONTAL FLOOR

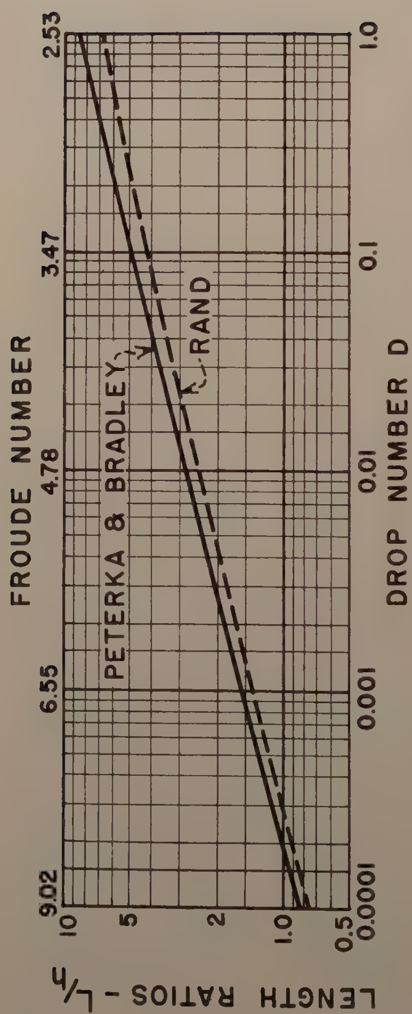

FIGURE 2. JUMP LENGTH FOR STRAIGHT DROP SPILLWAY

Discussion of
"ECONOMIC ASPECTS OF FLOOD PLAIN ZONING"

by H. W. Adams
(Proc. Paper 882)

H. ALDEN FOSTER, M. ASCE.[1]—Zoning of flood plains, as explained by Mr. Adams, involves the control of developments on any land located in the flood plain. It may have either of two basic purposes: (1) to protect the owner of the property under consideration from financial loss as a result of unwise construction, and (2) to prevent construction that might cause increases of flood damage in general in the flood plain.

Protection of the first type may require only a warning to the property owner as to the flood risks involved. If the owner decides to take a change of future loss, presumably that would be his privilege. On the other hand, if the owner tries to exert his rights by erecting structures that would constrict the flood channel and thereby raise the water level upstream during future floods, he would tend to nullify the beneficial results of protection by flood-control structures. Control of his property rights would then be justified in the public interest.

Zoning of the use of land in flood plains cannot be retroactive, so far as the erection of structures on the land is concerned. In this respect it is similar to municipal zoning regulations. After a building or other structure has once been erected in the flood plain, it can only be removed through governmental action after purchase by condemnation or by acquisition of an easement. In either case, the procedure would come under the classification of evacuation rather than zoning.

As Mr. Adams points out, the benefits obtained by zoning or evacuation are positive; if no structures are erected in the flood plain, no damage can result therefrom during future floods. Protection by flood-control structures, on the other hand, can seldom be complete except at excessive cost. Moreover, the estimated benefits from such protection are subject to the uncertainties of determination of flood frequencies. Where flood protection structures are erected by public agencies, the tendency is for owners of property in the natural flood plain to assume that their property is fully protected, without realization that a super-flood exceeding that for which the protective structures were designed may occur at some time in the future. More than one flood-protection project has had to be revised after experience has shown that the "maximum flood" assumed for its design has actually been exceeded.

It would undoubtedly be desirable to establish some system of zoning in certain flood plains even if no immediate program of flood protection is contemplated. The speaker knows of one such case where the flood plain was inundated to a depth of several feet by a flood that occurred in 1903. During the 52 years intervening since that date until this summer, no flood of comparable magnitude has occurred on this river. In the meantime, extensive

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real-estate developments have occurred in the flood plain. The property owners presumably do not have any knowledge of the flood history; or they may feel that the chance of such a large flood occurring in the future is very small. However, any hydrologist would admit that there is practically a certainty that a flood of equal magnitude, in volume of water discharged, will occur at some future time; and the damages resulting therefrom would be very much greater than those incurred in 1903, not only because of the greater value of property subject to flooding but also because of the obstruction to the flood discharge caused by the numerous buildings now located in the flood plain.

Undoubtedly, there are difficult questions involved in a situation such as that previously described, particularly as to what governmental authority should take the responsibility for warning potential property owners or for establishing zoning regulations for the flood plain. But if a damaging flood should occur in the future, there would undoubtedly be strenuous demands by the property owners that the State or Federal Government come to the rescue. Incidentally, there have been numerous examples of a somewhat similar situation in connection with property developments on ocean beaches. Legally, the principle of caveat emptor (let the buyer beware) may apply. But the average individual cannot be expected to have knowledge of the principles of hydrology that would warn him of potential danger. Advance study and warning by Federal or State agencies would doubtless result in appreciable financial savings to the general taxpayers in the future, although such a procedure certainly would not be favored by the parties interested in promoting new real-estate developments in the flood plains.

The previous discussion was prepared in June 1955. Since then we have experienced the unprecedented floods of August 18-20 and October 14-17 in the northeastern states. These floods have given fresh demonstration of the folly of building homes on the flood plain of a river, even if no flood has occurred at the given location for many years. The river valley previously described that was flooded in 1903 was again inundated in the recent October flood, though perhaps not as severely as before. Nevertheless much damage was done to private property, and numerous families had to be evacuated, according to newspaper reports.

A letter to the editor of a New Jersey newspaper on October 22 calls attention to the plight of these people. "These unfortunate people have lost all or have been mortally wounded through no fault of their own!" The question may be raised, however, as to whether the persons so affected were not to some extent responsible for their present losses, since they established their homes on the flood plain of their own free choice. A similar situation of a most poignant nature occurred on Broadhead Creek near East Stroudsburg, Penn., where many unfortunate people lost their lives at a camp located on the flood-plain of the stream, at a point that had not been flooded for many years.

Mr. Adams has rendered a service in pointing out some of the basic uses of flood-plain zoning. It is conceivable that a greater use of zoning, in combination with evacuation and partial protection by structures, may result in considerable economics in flood control operations by governmental agencies.

JOURNAL

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

ECONOMIC ASPECTS OF FLOOD PLAIN ZONING

H. W. Adams, M. ASCE[1]
(Proc. Paper 882)

SYNOPSIS

The zoning of flood plain lands for the alleviation of flood damages has not been extensively used as a primary means of damage prevention although there have been numerous applications of the principle of restriction to serve as a supplement to structural flood control measures. The economic aspects of zoning are essentially an alternative consideration to evacuation or to prevention.

INTRODUCTION

In the field of flood control planning, zoning might be considered as one leg of a triangle wherein the other legs would be evacuation and protection. The application of zoning restrictions normally are applied to an area prior to the time of appreciable economic development or at the time of redevelopment. When even minor development has taken place in a particular area, then evacuation as well as zoning must be considered as a means of damage prevention. Coupled with this is the third leg of the triangle, protection; since the alternative to evacuation is a consideration of the merits of providing protection in lieu of evacuation.

Some clarification may be in order with respect to the term "zoning." As used herein, it might be defined as a measure for preventing flood damage through control of buildings, or land uses, in areas that are subject to the flood hazard. Control of developments through regulation may range from the exclusion of all development that is subject to possible flood damages, to requirements for methods of building construction or to requirements that prospective purchasers of property be forewarned of the flood hazard. Zoning normally carries the exercise of police power for enforcement of the regulations.

The economics of flood plain zoning, therefore, requires consideration of

Note: Discussion open until June 1, 1956. Paper 882 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol 82, HY 1, February, 1956.

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essentially all facets of flood control planning, including an analysis of the flood characteristics and type of damages involved, feasibility of alternative measures for flood-damage prevention, and an economic determination of the relative value of zoning in relation to the other means of damage prevention.

Flood Characteristics and Damages

Any consideration of remedial flood prevention measures must start with the basic flood history of the particular locality and the several hydrologic determinations involved. This requires a hydrologic analysis and particularly the determination of frequency relations and the magnitude of maximum flood flows that may be anticipated. In the usual case, an unprotected area that is subject to frequent inundation is not subject to appreciable development, either for urban or agricultural uses. The duration of flooding may also be a factor in the consideration of restrictive regulations since communities located in headwater area that are subject to flashy type floods present one consideration as compared to that associated with the larger streams where areas may be flooded for long periods of time.

The development of damage data for economic analysis of proposed flood prevention measures would be based on the records of past flood occurrences at the particular locality. Damages normally would be expressed as average annual values as derived from the relation between the magnitude of damage and the frequency of occurrence.

Zoning in Relation to Other Measures

The inherent advantage of river valley lands as routes for transportation, industrial locations and fertility for agricultural production encourage their use in spite of the flood hazard; and discourage the application of zoning or limitations on use. While little use has been made of zoning as a primary damage-prevention measure there is a close relation in its use as a supplement to other measures.

In order for a particular area to develop, it must be relatively free from flooding. If the area is subjected to too frequent flooding, its state of development will be retarded so that the requirements for future uses of the area would dictate whether restrictive or preventative measures should be applied. This concept applies primarily to urban areas since the use of marginal agricultural lands for production of crops on a short term basis recognizes the risk involved before the venture is made. Decision on the method of damage prevention to be employed requires a prediction of the anticipated future uses of the area and consideration of alternative possibilities.

Restrictive controls on land and building use in flood plains is related to engineering structures in several ways. For example, protective measures may be considered the best and most feasible solution of a particular problem but a number of years may be required for accomplishment. Accordingly, restrictions may be applied as a temporary measure for the interim period. In another case, levees may be justified in providing only a partial degree of protection so that controls may be applied to regulate not only the type of building but the location, foundation, elevation of floor line, basement and other structural features in order to reduce the potential flood damages.

Zoning is usually associated with evacuation and operates to prevent future development after removal of the existing hazard. Zoning also may be adopted

to prevent the erection and repair of structures located in areas that are planned for ultimate evacuation on a long term basis. The costs of the evacuation program will thereby be reduced and the exposure of properties to the flood hazard is prevented.

The consequences of zoning should not be overlooked during planning studies when considered as a possible solution to a particular flood problem. When restrictions are correctly applied to an area in which the principle of zoning is clearly applicable, the results will be favorable. On the other hand, there are possible detrimental effects associated with zoning that must be carefully evaluated, such as, the stifling of free enterprise, the curb on growth and expansion of an area, and the negative approach to a problem as contrasted with positive damage prevention measures. These factors are difficult, if not impossible, of measurement in monetary terms.

States and municipalities in periods of expansion might well exercise more control over the use of undeveloped flood plains in order to avoid creating future flood problems.

Economic Principles

It is a basic requirement in flood control planning that the benefits to be derived from a proposed flood prevention measure must exceed the costs. Since zoning is applicable primarily to the less developed areas the magnitude of damages for current conditions would be small and require the "crystal ball" approach to evaluate the prospective developments. However, after the acceptance of the projected economic evaluation, it is then necessary to decide whether the potential development should be permitted by providing protection, or prevented by enforcement of a zoning regulation. Many complex and difficult problems are associated with the evaluation of flood prevention benefits connected with the ordinarily employed structural measures and those associated with zoning are considerably more complex because of the necessity for evaluation of intangible factors.

The basic concept of a benefit-cost analysis is to make a comparison of respective values on a comparable time base. Such comparison can be of average annual values, or of capitalized values for a specified period of time. Each determination requires application of a rate of interest and a period of time for amortization of the investment. The estimate of benefits accruing to the structural measures; levees, channels, storage, etc., are predicated on the basis of historical flood records assuming either the current or some reasonably anticipated state of development of the particular area under consideration. As a general rule, the future development of the area without the benefit of protection over the project life-time is conservatively estimated, perhaps on the order of 10 to 25 percent above existing conditions. In the case of zoning, a much greater extrapolation of future developments must be considered, possibly several times greater than that now existant, and this in itself may raise question as to the validity of the forecast. In contrast, if the future developments are considered relatively firm, one cannot ignore the economics of the alternative possibility of providing adequate flood protection if such measures are engineeringly feasible of accomplishment and the areas for future expansion of the community are limited.

There is a striking difference in the assurances of safety when comparisons are made between alternative measures of zoning and protection. Estimates of future flood frequencies and magnitudes are far from an exact

science, so that the anticipated degree of protection to be provided by structural measures cannot be positively assured. In addition, the operation and functional efficiency of structures are dependent upon human elements and these may fail in times of critical need. Zoning and the prevention of flood plain development does provide positive assurance against possible future flooding when such developments are located above all probable flood levels.

Corps of Engineers Planning

The Corps of Engineers, in the planning of comprehensive river basin programs for water resource development, gives consideration to evacuation and zoning as well as to other methods of damage prevention as a means of effecting flood control. Zoning as an exclusive method of flood control has not been extensively used by the Corps of Engineers, but such methods have been widely utilized as a supplement to other flood control measures.

Examples may be found in levee systems for a number of major river basins wherein the levees are set back from the river bank in order to provide a floodway for the accommodation of design flows. The lands located riverward of the levees are dedicated to the flood channel. In many cases, it is required that local authorities enact suitable zoning ordinances and give assurances against encroachment on the floodway capacity as a condition of local cooperation.

Adequate authority for modification of flood control projects already authorized by Congress is available to the Corps of Engineers. This authority stems from the Flood Control Act of 28 June 1938 and provides for evacuation of areas in lieu of constructing levees and floodwalls when found less costly. Provision also is made in this Act for financial assistance toward rehabilitation of the people evacuated. In addition, agreements may be entered into with States, local agencies, or the individuals concerned for reimbursement of expenditures incurred by them in accomplishing such evacuation and rehabilitation. Accordingly, adequate authority is available to the Corps of Engineers to undertake resettlement, or flood plain evacuation, as a primary flood damage prevention measure when found economically feasible to do so, although such authority has been used but little, since local residents are usually adverse to relocation if they can be protected.

Particular instances in which the concept of flood plain zoning, or evacuation, has been utilized by the Corps of Engineers in water resource planning are summarized as follows:

a) Essential features of the project for the Alluvial Valley of the Mississippi River are the by-pass channels and floodways required for accommodation of extraordinary flood flows. The right to use essentially all of these areas for floodway purposes has been obtained by purchase of flooding easements. This flowage right does not prevent the future development of the area and the flood damages that will occur when the floodways are used, although it serves the primary project purposes by imposing limitations on a portion of the area for the net benefit of the whole.

b) Planning for levee protection in a portion of the Wabash River basin has required the delineation of flood channels between levees located on opposite sides of the river and in different States. The Corps of Engineers' recommendation for authorization of these levees was predicated on the condition that the States enact legislation to designate and establish portions of the flood plain as floodways, as a condition of local cooperation. It also was required

that the States provide for acquisition of an estate or easement, to prevent future construction or improvements in the floodways which would cause any restriction of their carrying capacity.

c) A recent investigation of the flood problem at Odanah, Wisconsin resulted in recommendation for evacuation of the area, except school facilities, in lieu of providing other measures for flood damage prevention. The studies showed that moving the existing developments from the flood plain to high ground would be the most economical solution of the problem and, in addition, would provide a permanent solution. One provision of local cooperation was the requirement that local authorities prescribe and enforce regulations to prevent all future development and habitation in the evacuated area.

d) A number of pumping stations have been designed and constructed by the Corps of Engineers as an auxiliary facility for drainage of areas afforded protection by levees and floodways. Many of these stations have been designed with installed pumping capacities that are dependent upon the utilization of ponding areas for the temporary storage of storm run-off. Accordingly, any encroachment upon the areas reserved for ponding that causes a significant reduction in storage capacity will result in a degree of protection less than that adopted as a basis of design. In certain instances, only a minor reduction in available storage capacity may result in the installed pumping capacity being greatly deficient.

In order to assure that drainage facilities will provide the protection contemplated in the design, the Corps of Engineers usually requires that local interests procure flowage easements for ponding areas to prevent encroachment. In some cases, local interests are required to give satisfactory assurances that they will install the additional pumping capacity if the ponding areas are allowed to be developed for other purposes.

CONCLUSIONS

In summary, it may be stated that consideration should be given to the merits of flood plain zoning, as well as evacuation during the course of planning studies. In those instances where this method of effecting flood control is found least costly and to be a satisfactory solution of the problem, the method should be recommended. There is a problem as to how far the Federal Government can go in the effective utilization of zoning without infringing upon the rights of States and private individuals. There is also a problem on the sharing of costs between the Government and local interests for evacuation measures.

The many potential difficulties involved in prescribing and enforcing zoning and evacuation regulations must be weighed against the long-range requirements for development of an area and the effects thereof in possibly aggravating flood damages, increasing hazards to life and property, and future protection costs. Progress cannot be stopped but it can be guided wisely.

JOURNAL

HYDRAULICS DIVISION

Proceedings of the American Society of Civil Engineers

EVALUATING EFFECTS OF LAND-USE CHANGES ON SEDIMENT LOAD

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(Proc. Paper 886)

SYNOPSIS

The analytical method is given which was used in evaluating the effect of changes with time of cover density and land use upon sediment load characteristics of two tributary streams in the Tennessee Valley. The use of a time-regression function to represent the effect of changing cover and the statistical technique of Analysis of Variance are presented as a means of determining the significance of the continuous time trend in estimating the reduction of suspended sediment.

INTRODUCTION

In the Tennessee River basin, a number of areas have special resource and water problems which are important in the over-all conservation and utilization of soil and water resources of the Valley. Since its establishment in 1933, the Tennessee Valley Authority has included in its natural resources development program small watershed projects for studying land use and land management problems. Some projects were designed to determine the effects of different types of agricultural and forest covers and land management practices upon the hydrology of a watershed as well as upon the economic well-being of the prople using the land. The purpose of these projects is to promote basic research, develop methods of measuring and separating the effects of numerous variables, and explain, insofar as possible, the results obtained.

Other projects are concerned with the development of specific watersheds, presenting an opportunity for a demonstration of the TVA's applicable regional development programs integrated on a small watershed. Valuable data, as on runoff and sediment loads, are obtained which serve as measures for determining the effects of changes in cover and land-use practices.

Note: Discussion open until June 1, 1956. Paper 883 is part of the copyrighted Journal of the Hydraulics Division of the American Society of Civil Engineers, Vol. 82, HY 1, February, 1956.

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This paper demonstrates how the effects of land-use changes on suspended sediment load characteristics of two tributary watersheds were evaluated by the application of statistical methods of multiple regression.

The necessity of multiple regression becomes apparent when one considers that suspended sediment load in a stream varies widely from storm to storm. The variation is caused by the fluctuations of certain measurable hydrologic parameters and, in part, by unknown factors which produce random variations. These stormwise variations tend to obscure any change in the suspended sediment load which may be present due to change in cover density.

Initial trials by simple arithmetical methods or graphical correlations were unsuccessful in explaining the variations of the sediment load data. By these methods the most important factors producing these variations are considered successively, one at a time. It was expected that a multiple regression analysis might be more successful since it would explain the variation or "adjust" the data, to use the more common statistical term, by simultaneously removing the combined effects of the major causative factors. Residual variations in sediment load remaining after adjustment are much smaller than the variations in the original data.

White Hollow Watershed

The White Hollow watershed is a small area of 1715 acres located in eastern Tennessee and was acquired by the Tennessee Valley Authority in its land purchasing for Norris Reservoir. In 1934, about two-thirds of the watershed was in a poor quality forest having been subjected to frequent burning, woodland grazing, and cutting. Most of the remaining area was abandoned land with broom sedge cover, and only a small portion was being actively cultivated. In general, soils were severely eroded with numerous active gullies.

In the 18-year period between 1934 and 1952, annual precipitation over the watershed varied from 35.6 to 63.6 inches, averaging 46.6 inches. During this same period, annual runoff varied from 6.8 to 30.0 inches and averaged 18.3 inches.

The acquisition of this area by TVA and the forest protection and management then possible made this an ideal watershed for studying the effects of changes in vegetal cover upon runoff and sediment loads in an area taken out of agriculture and typical of the Appalachian Valley. The watershed management has included extensive erosion-control operations in 1934 and 1935 as well as tree plantations at that time and between 1938 and 1942. Natural reproduction was also an important factor in establishing the good cover which now exists.

The method of analysis used in avaluating the results of the 18-year record of sediment load investigations are given in this paper.

Measurements of precipitation, streamflow, and suspended sediment concentrations have been made in the watershed since 1935. Precipitation gages established on the watershed varied in number, at times, from one to seven recording gages. The relatively large number of gages was used in studies of minute variations in total rainfall and rainfall intensities. Actually, there was no important variation from gage to gage, and the microclimatic study showed that a single centrally-located rain gage was adequate for providing information on precipitation over the watershed. Therefore, the number of gages was reduced to one in June 1945. Streamflow from the watershed has

been measured by two installations. The original one consisted of a continuous recorder in a concrete gage house, the low-water control being a rectangular weir, 15.17 feet long and 0.4 foot deep set in a concrete dam across the creek. Flow was determined by the weir rating up to heads of 0.4 foot. Discharge measurements were made of flows above weir capacity, being taken upstream from the weir pool backwater.

As the character of the streamflow changed during the years the long, level crest of the weir no longer gave the desired sensitivity of measurement. In June 1948, the original installation was replaced by a modification of the Albany weir developed by the U. S. Geological Survey, the modification being necessary to increase the capacity. The gage house remained the same. The modified Albany weir was rated in the TVA Hydraulic Laboratory, making further current meter measurements at the site unnecessary.

From July 1935 to August 1949 suspended sediment was sampled manually. Thereafter, sampling has been accomplished by an automatic suspended sampler, taking an aliquot of $\frac{1}{105,000}$ part of the total flow. The automatic sampler was developed in the Hydraulic Laboratory of the Hydraulic Data Branch and was placed in operation at the White Creek measuring weir in August 1949. The device is designed to take a continuous sample of 1/100,000 of the total stream discharge and to store this sample in the concrete tank downstream. The sampling is accomplished in a series of steps, in each of which a proportional part of the discharge passing that step is retained and the balance wasted. A brass plate located in the measuring weir structure takes an initial sample of 1/100 of the total flow. At the next splitting point 1/10 of this reduced flow is retained, and this is followed by another divider which takes 1/2 of the remainder. At this point, the sample is directed to a pair of tipping buckets where the final splitting with retention of 1/50 of the flow at that point takes place just before the sample flows into the storage tank. The results of the laboratory testing showed the actual sampling ratio was 1/105,000.

At intervals, depending upon the amount of flow, the station is serviced and the sediment sample collected in the tank is measured. This measurement involves the determination of the volume of water and sediment in the tank. The tank contents are thoroughly stirred and representative pint-sized samples taken. The sediment contents of these samples are determined by analysis in the laboratory.

With the installation of the modified Albany weir and the sediment sampler the station has been made practically automatic and independent of the local observer problem.

Observations have been made to confirm that no significant measurable difference exists between the two methods of sampling. Manual samples are taken about weekly when servicing the sampler as a check upon the functioning of the equipment. From the streamflow data and samples of sediment concentration it was possible to estimate the suspended sediment load for selected storm periods. The sediment load was an index of the effectiveness of increasing forest cover and resultant litter in controlling soil erosion.

Method of Analysis

The data available for analysis consisted of suspended sediment loads for 145 periods of time. These periods were essentially storm-runoff occurrences and were distributed throughout the 18 years of record. For the

period during which sediment was sampled manually all storms were included in the analysis for which the sampling was considered adequate to draw a continuous curve of sediment concentration. For the period during which sediment was sampled by the automatic device, all sampler periods were included which had sampling errors of less than twenty percent variation from the calibration sampling ratio. These sampling periods varied from a few days during high discharge to about two weeks during extremely low discharges.

The analysis of the sediment data from the White Hollow watershed was designed to evaluate an average time-trend in the reduction of suspended sediment loads. All data were used to develop a single equation to estimate this continuous time trend by the method of least squares. This procedure allows efficient use of all the data in determination of one set of regression coefficients, and also allows significance tests based on the analysis of variance associated with the regression parameters.

Measurements of forest cover were made at 5-year intervals and were, therefore, too infrequent for their direct use in a regression equation. It was necessary to assume that the effect of changing cover on sediment production was some function of time.

The measurements of forest cover were found to be a substantial verification of a standard law of growth, in which it is stated that the rate of increase in density is proportional to the difference between the existing density and its final value. This law can be expressed by the differential equation:

$$\frac{dD}{dT} = b\,(D_1 - D) \tag{1}$$

where dD = differential of cover density

dT = differential of time

D = density at any time T

D_1 = final value of the density

and b = proportionality constant

When this equation is integrated and solved for D there results

$$D = D_1 - ce^{-bT} \tag{2}$$

where c = constant of integration

and e = base of the natural logarithms

Since the cover density increases to some final value by an exponential function of time its effect upon the production of sediment was assumed to be a decreasing exponential function of time. The term e^{-bT} was chosen for this function. The assumed negative sign of the exponent required confirmation by the least-squares solution.

In addition to the time-regression function it is necessary to include in the complete regression equation all those hydrologic variables which are known to have significant effect in the production of storm sediment loads. Since the watershed cover has become entirely forest, the major variations in sediment load from storm to storm must be associated with these basic hydrologic variables.

After preliminary testing of various combinations of parameters the regression model selected was:

$$Y = a e^{-bT} P^{d} H^{f} K^{g} Q^{h} \tag{3}$$

where Y = storm sediment load in tons
T = time by water years (1935-1936 = 1, etc.)
P = storm rainfall in inches
H = storm rainfall duration in hours
K = average temperature in degrees Fahrenheit for 10 days prior to the storm
Q = peak discharge of total flow in cubic feet per second

and a, b, d, f, g, and h are constants of the equation

Equation 3 is not linear in the constants to be determined and, therefore, cannot be evaluated by the method of least squares. The equation can be transformed by taking logarithms of both sides.

$$\text{Log } Y = A - mT + d \text{ Log } P + f \text{ Log } H + g \text{ Log } K + h \text{ Log } Q \tag{4}$$

where A = Log a
m = b Log e = 0.4343b

and A, m, d, f, g, and h are the constants to be evaluated.

In this form the regression coefficients can be evaluated by fitting the equation to the data by the method of least squares. This fitting, of course, minimizes the square of the residual errors of the logarithms of the sediment load.

Equation 4 was fitted to the data with and without the time-regression function in order to test the statistical significance of this factor. The difference in the sums of squares caused by omitting this variable from the equation was used in making the standard analysis of variance test for significance..

The numerical results of fitting to the data are shown in Tables 1 and 2. The multiple correlation coefficient for the complete equation was 0.855, indicating the high degree of adjustment of the sediment data which was accomplished by the chosen regression model. The value of the statistic known as "F" associated with the time regression term was computed as 63.2. The published critical value of the statistic "F" for a one percent level of significance, is 6.81. Since the computed "F" value for the time function is much larger than the critical value at the one percent level the contribution of this term to the equation is judged statistically significant.

The statistic "F" was also computed for each of the hydrologic parameters in Equation 4. These values are included in Table 2.

In Figure 1 is shown the computed time-regression function. This curve gives the decrease with time of the sediment load for an "average" storm. The "average" storm was arbitrarily defined by holding the four storm parameters constant at values corresponding to the mean values of their logarithms. These values were 1.49 inches for rainfall, 12.8 hours for duration, 55.3 degrees Fahrenheit for temperature, and 8.7 cubic feet per second for peak discharge. T was assigned values from 1 to 17 for the

water years 1935-1936 to 1951-1952. Values of sediment load Y were computed for each year for the "average" storm. The reduction in sediment load from 1936 to 1952 was 6.8 tons. This reduction was 93 percent of the 1936 value.

There is no simple way to present graphically the individual contributions of the four storm parameters to sediment production. Because the four variables are interrelated, no real physical meaning can be attached to variations in one of the variables with the other three being held constant. Sediment loads can be computed for various logical combinations of the variables.

In Figure 2 is a family of curves showing the relation between summertime storm rainfall and sediment load for several years on the time-regression function. The long-term average temperature for July of 76 degrees was substituted for the 10-day average temperature in computing curves for the summer season. Rainfall duration was assumed to be 5 hours. The peak discharges for the summer season were determined from previously established relationships between rainfall and discharge during the period of increasing cover density.

Chestuee Creek Watershed

The Chestuee Creek watershed includes the drainage of 134 square miles or about 85,000 acres of mixed agricultural and forest lands located in the east central portion of the Tennessee Valley. The creek runs in a general southwesterly direction, draining into the Hiwassee River arm of the Chickamauga Reservoir.

During the 10-year period between 1944 and 1954, annual precipitation over the watershed varied from 43.5 to 61.8 inches, averaging 53.9 inches while annual runoff varied from 16.6 inches to 28.9 inches, averaging 23.0 inches.

The stream pattern consists of one main creek, Chestuee, with one major and nearly parallel tributary, Middle Creek, in the upper end of the watershed. Numerous small creeks drain the higher lands into Chestuee and Middle Creeks. The soils have been formed from sandstone, shale, and limestone. As a result of poor land-use practices in the past, both main streams and tributaries have small capacity because of the filling of the channels by material eroded and transported from the uplands.

The TVA recognized the necessity and desirability of determining and adopting a policy with respect to the solution of water control problems on rural watersheds within the Tennessee Valley which are individually of less than regional significance. The Chestuee Creek watershed offered a good opportunity for determining remedial measures which may serve as a guide for the solution of the general problem in the Valley.

Measurements of precipitation, streamflow, and suspended sediment concentrations have been made on several subdivisions of the watershed since 1944. Precipitation has been measured by 5 recording and 16 nonrecording gages located to cover the watershed and its subdivisions adequately. Streamflow was measured at six stations equipped with continuous water level recorders. Discharge measurements were taken at each station with a current meter on a routine schedule supplemented by additional measurements during the rise, crest, and recession of increases in flow. From these measurements, discharge rating curves were developed to establish

current stage-discharge relations at each station. Suspended sediment concentrations were sampled manually at the six established stream gaging stations. In addition to a routine weekly sampling schedule, samples were taken of increases in discharge following heavy rainfall during the rise, the crest, and the recession of flow. During the first nine years, a horizontal trap type sampler was used. Thereafter, this sampler was replaced by a depth-integrating sampler.

The daily sediment loads were computed from continuous plottings of streamflow and sediment concentrations. The monthly sediment loads were obtained by a summation of daily loads. These monthly loads were divided by the drainage area above the respective station to obtain the monthly load per square mile as used in the analysis. As in the White Hollow experimental watershed investigations the suspended sediment load was chosen as one index to investigate the effect of changing cover density and land-use management upon soil erosion. Only the results for the entire watershed above the lowermost gaging station are presented in this paper.

Two land-use surveys were made in the Chestuee Creek watershed, one in 1944 at the beginning of hydrologic investigations and another in 1954 to determine the change in character of cover that had taken place in those ten years. The TVA Hydraulic Data Branch prepared a land-use classification which was used in each survey and had particular application to infiltration and erosion characteristics of the land. This classification divided all land into three principal groups, cultivated crops, pasture, and forest with further subdivision of each group depending upon the quality of the cover with regard to infiltration and erosion. The results of these surveys showed an improvement in the watershed cover in the ten-year period. It is logically assumed that this is the most probable cause of the progressive reductions noted in sediment loads carried by the streams.

Method of Analysis

Suspended sediment loads were computed from observational data for each month during the ten-year period from January 1944 through December 1953. These 120 monthly load values were analyzed for the total period of record to obtain a continuous time function for sediment change. No attempt was made to separate the data into partial periods for comparative analysis.

The Chestuee Creek subwatersheds have mixed covers of crop land, pasture, and forest. Any particular parcel of land may change from one cover classification to another at the discretion of the owner. Obviously, no simple time-regression function can be rigorously demonstrated as describing a physical law on such a complex area. Basically, however, the function should show an increase or decrease in the average sediment load, and an increasing or decreasing rate of change. Furthermore, the function should be capable of approaching some constant value. An exponential function as an additive term in a multiple regression equation will, in general, meet these requirements.

Use of an exponential function as an additive term presents some difficulty since the exponent cannot be evaluated directly by the method of least squares. Consequently, an abbreviated equation was used to evaluate the exponent on a trial-and-error basis.

The regression equation used for the Chestuee watershed data is empirical. Selection of the specific exponent for each variate was made by examination of portions of the data in which the effect of all other variates was small.

For example, the effect of rainfall on sediment load was studied for similar seasons of the year which also had similar antecedent moisture conditions. The smallest simple integer, for power or root, which appeared to express adequately the effect of that variate upon sediment load was chosen as the exponent of that variate. Little is known concerning the theoretical regression model expressing the effects of the various hydrologic parameters integrated over a complex combination of land-use practices and vegetal cover.

The abbreviated regression equation was:

$$Y = a + c e^{-bT} + d P^2 \sqrt[3]{P_1}\,(2 + \sin M) \qquad (5)$$

where
- Y = monthly sediment load in tons per square mile
- T = time by months (January 1944 = 0.01, February 1944 = 0.02, etc.)
- P = rainfall for the current month in inches
- P_1 = rainfall for the previous month in inches
- M = season by months, evaluated by arbitrarily setting January = 30°, February = 60°, etc.
- e = base of the natural logarithms

and a, b, c, and d are constants to be evaluated

In order to determine the best value for the exponent b, various values were assumed. The abbreviated equation was fitted to the data by least squares, and the residual error of the regression was computed. This was repeated for several values of b. The trial values of b were plotted against the residual errors resulting from their use. This process was continued until the value of b which gave the minimum residual error could be determined from the plot. This plot also confirmed the assumed negative sign of the exponent b.

After the determination of the best value of b, in the abbreviated equation, a more complete equation was fitted to the data. This equation was:

$$Y = a + c e^{-bT} + d P^2 \sqrt[3]{P_1}\,(2 + \sin M) + f P^2 \sqrt[3]{P_1} \qquad (6)$$

where the terms are as previously defined and f is an additional constant to be determined.

Equation 6 is the form used in fitting by least squares to estimate values of a, c, d, and f. That value for b which was determined in the abbreviated equation was used in Equation 6. The sense of the equation is better understood by rearranging terms and putting the equation in the form:

$$Y = a + c e^{-bT} + d P^2 \sqrt[3]{P_1}\,\sin M + (f + 2d)\, P^2 \sqrt[3]{P_1} \qquad (7)$$

The equation thus contains the exponential time function, a cross-product of a rainfall term and a seasonal term, and the rainfall term without modification by season. If the coefficient f is small in comparison to the value 2d, as was found in this case, Equation 7 is essentially the same as the abbreviated Equation 5. The numerical results of fitting to the data are shown in Tables 1 and 3. Fitting Equation 6 to the data produced a multiple correlation coefficient of 0.896, showing the high degree of adjustment.

Figure 3 shows the computed time-regression function for the average monthly sediment load. In the 10-year period since 1944 the reduction in the average monthly sediment loads was 48 percent.

The statistical significance of the time-regression function was determined by analysis of variance, the same procedure used in the analysis of the White Hollow data. The computed value of "F" was 10.97 for the time term. As this value is larger than the critical value of 6.90 at the one percent significance level, the time function e^{-bT} is judged statistically significant. The computed values for the statistic "F" for the time term and other parameters in Equation 6 are shown in Table 3.

ACKNOWLEDGMENTS

The investigations and analyses were made under the general direction of Albert S. Fry, Chief, Hydraulic Data Branch, as a portion of the work in the Hydrology Section. The writers acknowledge the assistance of Dr. G. Eugene Albert, Associate Professor of Mathematics, University of Tennessee, and consultant to the Hydraulic Data Branch on the application of statistical techniques in the analysis of hydrologic data.

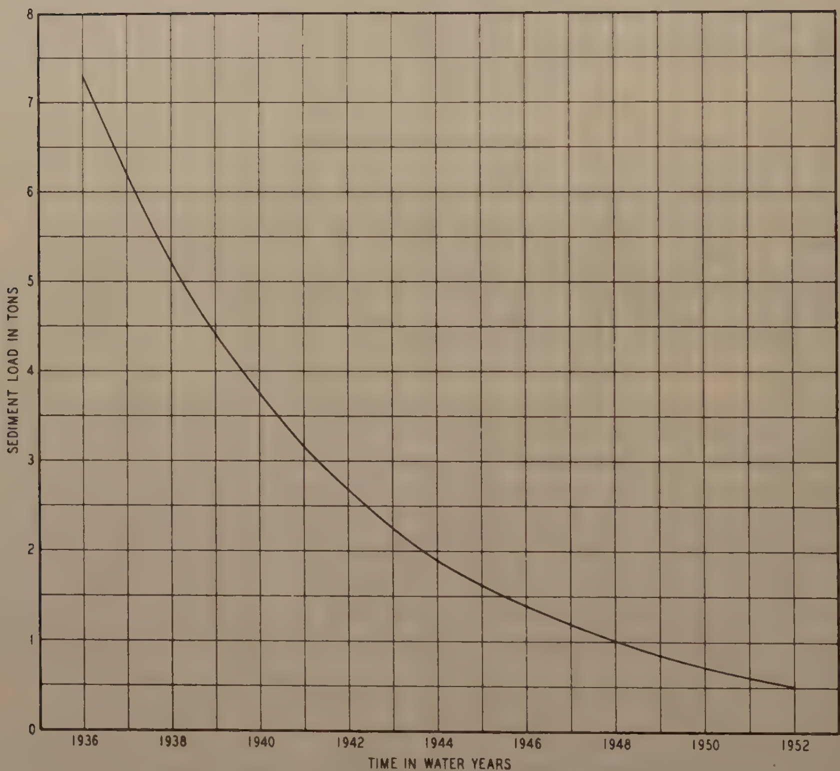

FIGURE I. WHITE HOLLOW WATERSHED SEDIMENT LOAD FOR AN AVERAGE STORM REGRESSION WITH TIME

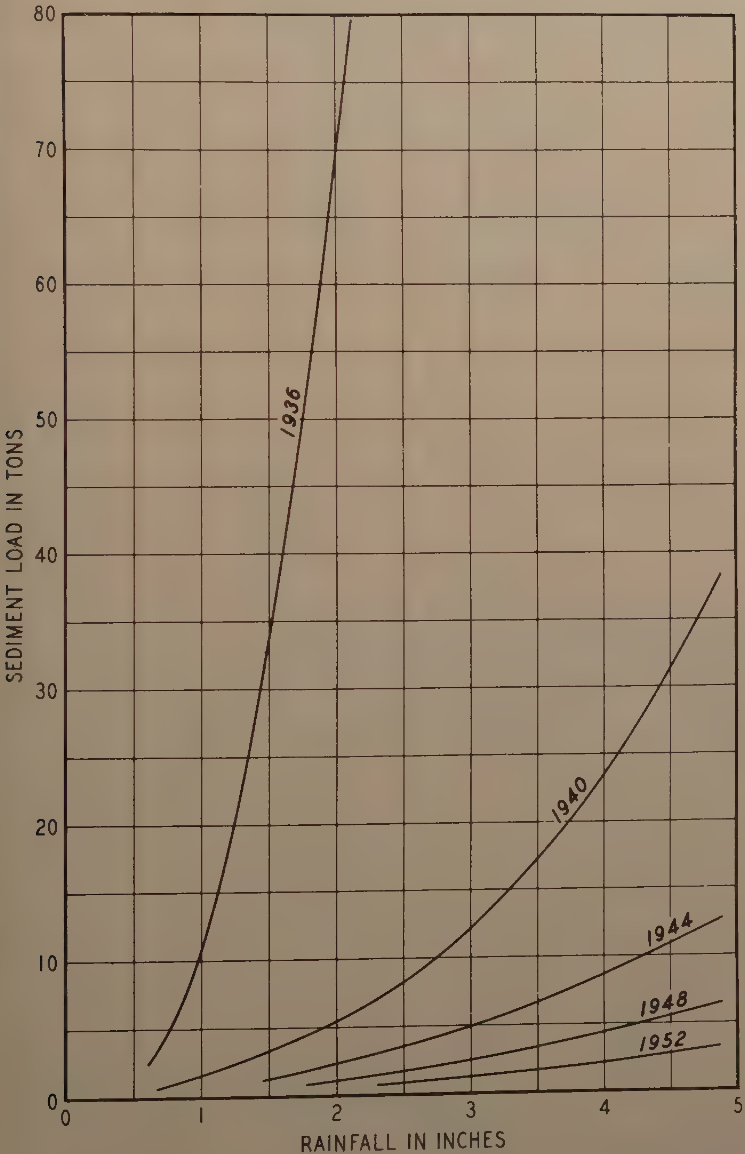

FIGURE 2. WHITE HOLLOW WATERSHED
SEDIMENT LOAD VS. SUMMER STORM RAINFALL

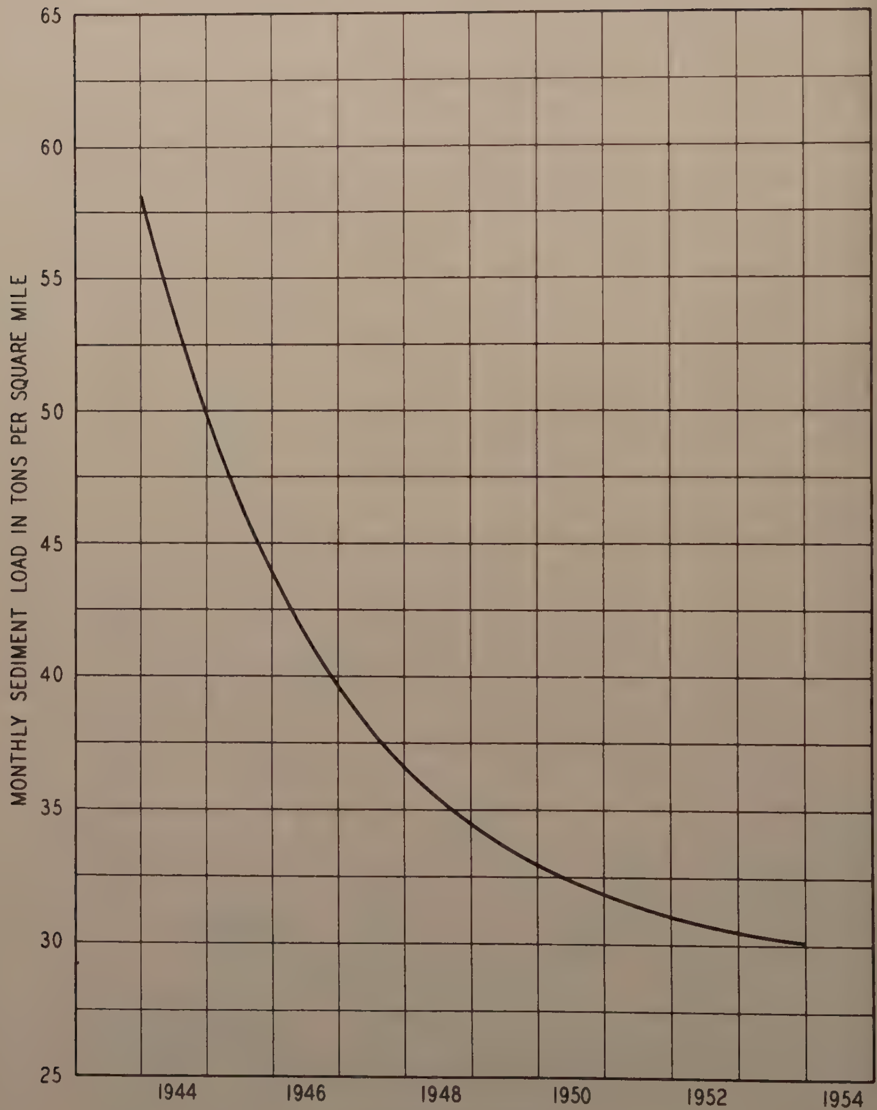

FIGURE 3. CHESTUEE CREEK WATERSHED AVERAGE MONTHLY SEDIMENT LOAD REGRESSION WITH TIME

TABLE 1

RESULTS OF ADJUSTMENT BY MULTIPLE REGRESSION

	White Hollow	Chestuee Creek
Size of Sample	145	120
Total Sum of Squares	112.49	361322
Sum of Squares Explained by the Regression Model	82.24	289728
Residual Sum of Squares	30.25	71594
Coefficient of Determination, R^2	0.731	0.802
Correlation Coefficient, R	0.855	0.896
Error Variance, s^2	0.218	617.2
Standard Error, s	0.467	24.84

TABLE 2

WHITE HOLLOW WATERSHED

ESTIMATED VALUES OF THE REGRESSION COEFFICIENTS

Variable	Symbol	Associated Coefficient Symbol	Value	Statistic "F"
		a	52.4	--
Time	T	-b	0.167	63.2
Rainfall	P	d	0.93	7.06
Duration	H	f	-0.38	3.61
Average Temperature	K	g	-0.78	3.00
Peak Discharge	Q	h	0.89	65.4

TABLE 3

CHESTUEE CREEK WATERSHED

ESTIMATED VALUES OF THE REGRESSION COEFFICIENTS

Variable	Symbol	Associated Coefficient Symbol	Value	Statistic "F"
		a	-8.70	--
Time	$e^{-2.8T}$	c	29.88	10.97
Rainfall and Season	$P^2 \sqrt[3]{P_1}$ Sin M	d	0.433	49.0
Rainfall	$P^2 \sqrt[3]{P_1}$	f + 2d	0.833	231.1

PROCEEDINGS PAPERS

technical papers published in the past year are identified by number below. Technical-n sponsorship is indicated by an abbreviation at the end of each Paper Number, the s referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering nics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), y Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and g (SU), and Waterways (WW) divisions. Papers sponsored by the Board of Direction are ed by the symbols (BD). For titles and order coupons, refer to the appropriate issue of Engineering." Beginning with Volume 82 (January 1956) papers were published in Jour- the various Technical Divisions. To locate papers in the Journals, the symbols after per numbers are followed by a numeral designating the issue of a particular Journal in the paper appeared. For example, Paper 861 is identified as 861 (SM1) which indicates paper is contained in issue 1 of the Journal of the Soil Mechanics and Foundations Divi-

VOLUME 81 (1955)

JARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), (WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)c, 622(IR), 623(IR), 624(HY)c, 625(HY), (HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

H: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)c, 642(SA), (SA), 644(SA), 645(SA), 646(SA), 647(SA)c, 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), (ST), 654(ST)c, 655(SA), 656(SM)c, 657(SM)c, 658(SM)c.

: 659(ST), 660(ST), 661(ST)c, 662(ST), 663(ST), 664(ST)c, 665(HY)c, 666(HY), 667(HY), (HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), (HY).

679(ST), 680(ST), 681(ST), 682(ST)c, 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), (SA)c, 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), (PO)c, 700(PO), 701(ST)c.

: 702(HW), 703(HW), 704(HW)c, 705(IR), 706(IR), 707(IR), 708(IR), 709(HY)c, 710(CP), (CP), 712(CP), 713(CP)c, 714(HY), 715(HY), 716(HY), 717(HY), 718(SM)c, 719(HY)c, (AT), 721(AT), 722(SU), 723(WW), 724(WW), 725(WW), 726(WW)c, 727(WW), 728(IR), (IR), 730(SU)c, 731(SU).

: 732(ST), 733(ST), 734(ST), 735(ST), 736(ST), 737(PO), 738(PO), 739(PO), 740(PO), (PO), 742(PO), 743(HY), 744(HY), 745(HY), 746(HY), 747(HY), 748(HY)c, 749(SA), 750(SA), (SA), 752(SA)c, 753(SM), 754(SM), 755(SM), 756(SM), 757(SM), 758(CO)c, 759(SM)c, (WW)c.

ST: 761(BD), 762(ST), 763(ST), 764(ST), 765(ST)c, 766(CP), 767(CP), 768(CP), 769(CP), (CP), 771(EM), 772(EM), 773(SA), 774(EM), 775(EM), 776(EM)c, 777(AT), 778(AT), (SA), 780(SA), 781(SA), 782(SA)c, 783(HW), 784(HW), 785(CP), 786(ST).

EMBER: 787(PO), 788(IR), 789(HY), 790(HY), 791(HY), 792(HY), 793(HY), 794(HY)c, (EM), 796(EM), 797(EM), 798(EM), 799(EM)c, 800(WW), 801(WW), 802(WW), 803(WW), (WW), 805(WW), 806(HY), 807(PO)c, 808(IR)c.

BER: 809(ST), 810(HW)c, 811(ST), 812(ST)c, 813(ST)c, 814(EM), 815(EM), 816(EM), 7(EM), 818(EM), 819(EM)c, 820(SA), 821(SA), 822(SA)c, 823(HW), 824(HW).

MBER: 825(ST), 826(HY), 827(ST), 828(ST), 829(ST), 830(ST), 831(ST)c, 832(CP), 3(CP), 834(CP), 835(CP)c, 836(HY), 837(HY), 838(HY), 839(HY), 840(HY), 841(HY)c.

MBER: 842(SM), 843(SM)c, 844(SU), 845(SU)c, 846(SA), 847(SA), 848(SA)c, 849(ST)c, 0(ST), 851(ST), 852(ST), 853(ST), 854(CO), 855(CO), 856(CO)c, 857(SU), 858(BD), 859(BD), 0(BD).

VOLUME 82 (1956)

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